

Mathematics and Music  
2007 REU  
Abstract  
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Have you ever wondered if the relationship between mathematics and music extends to the composition and interpretation of musical works? If you have, then this course is for you. If you already know the answer because you participated last year, then this course is still for you because we will have new material.

A central concern of music theory is to find a good way of hearing a piece of music and to communicate that way of hearing. Music theorists often draw upon mathematics to create conceptual categories towards this end. In recent years, basic tools from group theory, combinatorics, and topology have entered the realm of musical analysis.

We will discuss several currents in modern mathematical music theory, including David Lewin's Transformational Theory and Guerino Mazzola's Topos of Triads. Lewin's theory asks: which transformations are idiomatic for a work of music? For example, any fugue contains transpositions and inversions of the subject, and recognizing this pattern makes a fugue more enjoyable for both listener and performer. Another instance of a transformation assigns to a major chord its relative minor, something that we hear on the radio every day. Lewin applied the theory of groups and group actions to great effect in his analyses, and we will recount how in various musical examples. Group theory is used both as a language and as a vehicle for musical insight.

Mazzola's Topos of Triads, on the other hand, builds on Grothendieck's notion of topos. Topos theory has found application in algebraic geometry, logic, and category theory. It is a great surprise that such a sophisticated notion has anything to do with music at all! We will attempt to understand this pioneering work in elementary terms.

We will also discuss Scale Theory, Set Theory, and Dmitri Tymoczko's Geometry of Musical Chords, which visualizes voice leadings as paths on an orbifold.

Familiarity with any notions mentioned above is not necessary for this module. Nor is the ability to read music a prerequisite, since we aim to see and hear mathematics in action.