Some Characteristic Morphisms of Triadic Monoid Subactions

Recall: If A is a subobject of X, its characteristic morphism $\chi_A : X \longrightarrow \Omega$ is defined by

$$\chi_A(x) := \{h \in \mathcal{T} | hx \in A\}.$$

$j_{\mathcal{P}}, j_{\mathcal{L}}, j_{\mathcal{R}} : \Omega \longrightarrow \Omega$ are the characteristic morphisms of the respective subobjects $\{\mathcal{P}, \mathcal{T}\}, \{\mathcal{L}, \mathcal{P}, \mathcal{T}\}, \{\mathcal{R}, \mathcal{P}, \mathcal{T}\}$ of Ω.

x	$j_{\mathcal{P}}(x)$	$j_{\mathcal{L}}(x)$	$j_{\mathcal{R}}(x)$
$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$
$\mathcal{C}$	$\mathcal{C}$	$\mathcal{R}$	$\mathcal{L}$
$\mathcal{L}$	$\mathcal{L}$	$\mathcal{T}$	$\mathcal{L}$
$\mathcal{R}$	$\mathcal{R}$	$\mathcal{R}$	$\mathcal{T}$
$\mathcal{P}$	$\mathcal{T}$	$\mathcal{T}$	$\mathcal{T}$
$\mathcal{T}$	$\mathcal{T}$	$\mathcal{T}$	$\mathcal{T}$

x	$\chi_{\{0,1,4\}}(x)$
0	$\mathcal{T}$
1	$\mathcal{T}$
2	$\mathcal{C}$
3	$\mathcal{R}$
4	$\mathcal{T}$
5	$\mathcal{L}$
6	$\mathcal{R}$
7	$\mathcal{R}$
8	$\mathcal{L}$
9	$\mathcal{P}$
10	$\mathcal{R}$
11	$\mathcal{C}$

$\chi_{\{0,1,4\}} : \mathbb{Z}_{12} \longrightarrow \Omega$ is the characteristic morphism of the subobject $\{0, 1, 4\}$ of $\mu[1, 4] : \mathcal{T} \times \mathbb{Z}_{12} \longrightarrow \mathbb{Z}_{12}$.

More Characteristic Morphisms

Proposition 1 *Recall that* $\mu[m,n]$ *denotes the action of* $\mathcal{T}$ *on* $\mathbb{Z}_{12}$ *where the generators* f *and* g *act as* ${}^{m}3$ *and* ${}^{n}8$ *respectively. Then the action* $T_k \circ \mu[m,n]$ *is the same as* $\mu[m-2k, n-7k]$*. Further, if* A *is a subobject of* $\mathbb{Z}_{12}$ *under the action* $\mu[m,n]$*, then* T_kA *is a subobject of* $\mathbb{Z}_{12}$ *under the action* $T_k \circ \mu[m,n]$ *and* $\chi_{T_kA \subseteq T_k \circ \mu[m,n]} = \chi_{A \subseteq \mu[m,n]} \circ T_{-k}$.

$\chi_{\{0,10,4\}} : \mathbb{Z}_{12} \longrightarrow \Omega$ is the characteristic morphism of the subobject $\{0,4,10\}$ of $\mu[10,4] : \mathcal{T} \times \mathbb{Z}_{12} \longrightarrow \mathbb{Z}_{12}$.

x	$\chi_{\{0,10,4\}}(x)$	$\chi_{\{1,11,5\}}(x)$	$\chi_{\{3,1,7\}}(x)$
$0{=}C$	$\mathcal{T}$	$\mathcal{C}$	$\mathcal{R}$
$1{=}G$	$\mathcal{R}$	$\mathcal{T}$	$\mathcal{T}$
$2{=}D$	$\mathcal{L}$	$\mathcal{R}$	$\mathcal{C}$
$3{=}A$	$\mathcal{R}$	$\mathcal{L}$	$\mathcal{T}$
$4{=}E$	$\mathcal{T}$	$\mathcal{R}$	$\mathcal{R}$
$5{=}B$	$\mathcal{C}$	$\mathcal{T}$	$\mathcal{L}$
$6{=}F\sharp$	$\mathcal{P}$	$\mathcal{C}$	$\mathcal{R}$
$7{=}C\sharp$	$\mathcal{R}$	$\mathcal{P}$	$\mathcal{T}$
$8{=}G\sharp$	$\mathcal{L}$	$\mathcal{R}$	$\mathcal{C}$
$9{=}E\flat$	$\mathcal{R}$	$\mathcal{L}$	$\mathcal{P}$
$10{=}B\flat$	$\mathcal{T}$	$\mathcal{R}$	$\mathcal{R}$
$11{=}F$	$\mathcal{C}$	$\mathcal{T}$	$\mathcal{L}$

Interesting Morphisms $\mathbb{Z}_{12} \longrightarrow \Omega$ and the Subobjects They Classify

$\{0, 1, 4\} = \{C, G, E\}$

$j_{\mathcal{P}} \circ \chi_{\{0,1,4\}}$	$\{0, 1, 4, 9\}$	major-minor mix
$j_{\mathcal{L}} \circ \chi_{\{0,1,4\}}$	$\{0, 1, 4, 5, 8, 9\}$	hexatonic system
$j_{\mathcal{R}} \circ \chi_{\{0,1,4\}}$	$\{0, 1, 3, 4, 6, 7, 9, 10\}$	octatonic system

$\{0, 10, 4\} = \{C, B\flat, E\}$

$j_{\mathcal{P}} \circ \chi_{\{0,10,4\}}$	$\{0, 4, 6, 10\}$	french augmented sixth
$j_{\mathcal{L}} \circ \chi_{\{0,10,4\}}$	$\{0, 2, 4, 6, 8, 10\}$	even wholetone system
$j_{\mathcal{R}} \circ \chi_{\{0,10,4\}}$	$\{0, 1, 3, 4, 6, 7, 9, 10\}$	octatonic system

$T_3\{0, 10, 4\} = \{3, 1, 7\} = \{A, G, D\flat\}$

$j_{\mathcal{P}} \circ \chi_{\{3,1,7\}}$	$\{3, 7, 9, 1\}$	french augmented sixth
$j_{\mathcal{L}} \circ \chi_{\{3,1,7\}}$	$\{3, 5, 7, 9, 11, 1\}$	odd wholetone system
$j_{\mathcal{R}} \circ \chi_{\{3,1,7\}}$	$\{3, 4, 6, 7, 9, 10, 0, 1\}$	octatonic system

The respective triad is considered to be "locally present" if any of the three respective sets is present.