The Geometry of Musical Chords

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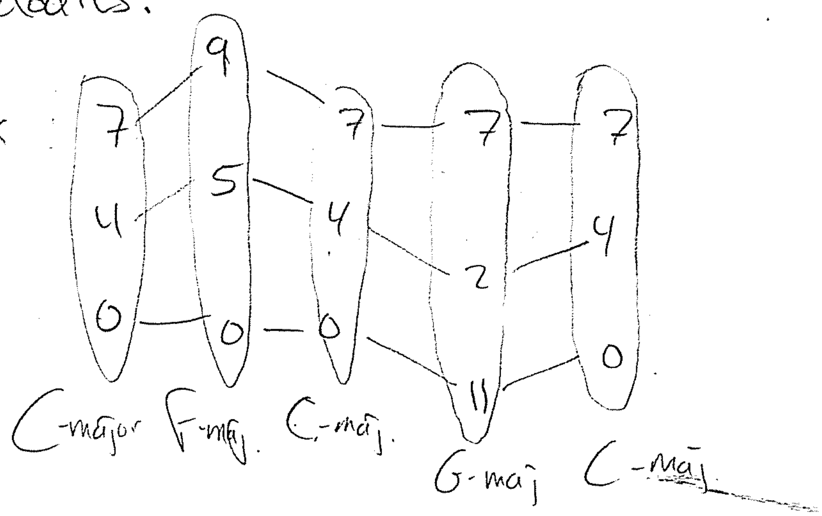
Talk by Thomas Fiore

In this talk we describe a way of visualizing voice leadings on an orbifold, and watch some movies. This introduction to voice leading will smoothly lead into neo-Riemannian theory.

Voice Leading

This is the technique of connecting notes in a series of chords to form simultaneous melodies.

Ex:



Conventions: - voices do not all move in same direction by same amount (independent)

- voices move in very short distances

- voice paths do not cross.

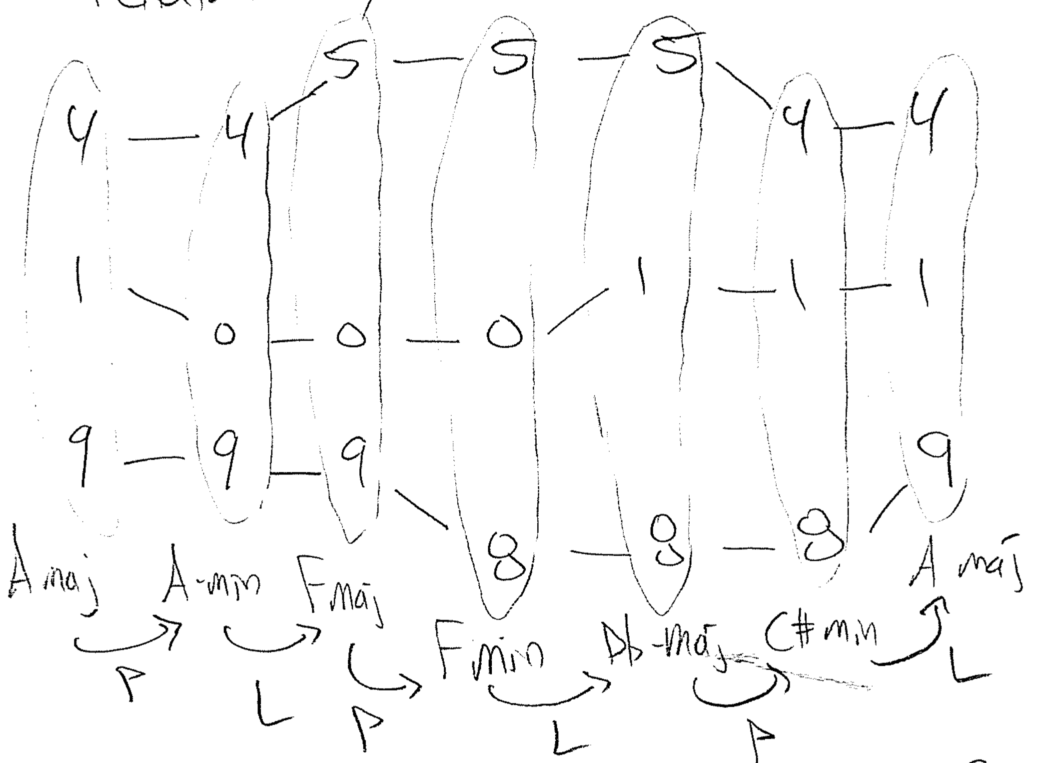
All of these conditions are satisfied in the example above.

Generally, one is interested in voice leadings between transpositionally or inversionally related chords.
A chord admits such voice leading only if it is nearly symmetrical under transposition, inversion, or permutation.

Rem In the example, all chords are related by transposition, and 0,4,7 is nearly symmetrical under an inversion

$$I_7(\{0,4,7\}) = \{7,3,0\}.$$

Ex Hexatonic System of major-minor triads of Cohn.



Check the conventions above, and general conditions

$\{0,1,4,5,8,9\}$

We've seen this in the work of Noll, but originates w/ Cohn.

Ex C-major scale $\longrightarrow$ G-major scale

$F \longmapsto F\#$

(and so on in circle of fifths).

Question: Given two chords how do you find the minimal voice leading? Can be programmed in polynomial time. (Tymoczko).

Manifolds and Orbifolds

Def A manifold is a topological space that locally looks like $\mathbb{R}^n$ for some fixed n.

Ex $\mathbb{R}^n$, S^1 = unit circle, S^n = unit ball, $S^1 \times S^1$ = torus.

Def An orbifold is a topological space that locally looks like a manifold modulo the smooth action of a finite group.

Ex $\mathbb{R}/\{\pm 1\}$ where -1 acts by $x \longmapsto -x$.

Ex $\left(\mathbb{R}/12\mathbb{Z}\right)^n / Sym(n)$ where $Sym(n)$ = symmetric grp on $\{1,..,n\}$.

$$\sigma(x_1,..,x_n) := (x_{\sigma^{-1}1}, x_{\sigma^{-1}2}, .. x_{\sigma^{-1}n})$$

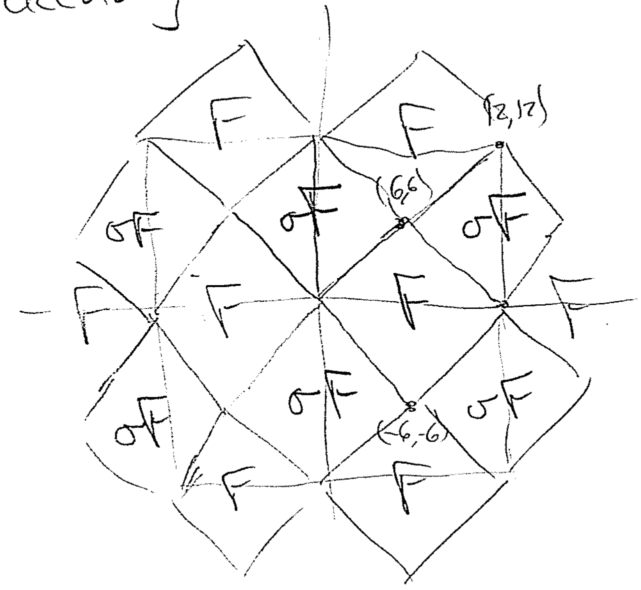
$\left(\mathbb{R}/12\mathbb{Z}\right)^n / Sym(n)$ = set of unordered n-element subsets of $\mathbb{R}/12\mathbb{Z}$ where repeats are allowed.

= orbifold of chords with n pitch classes

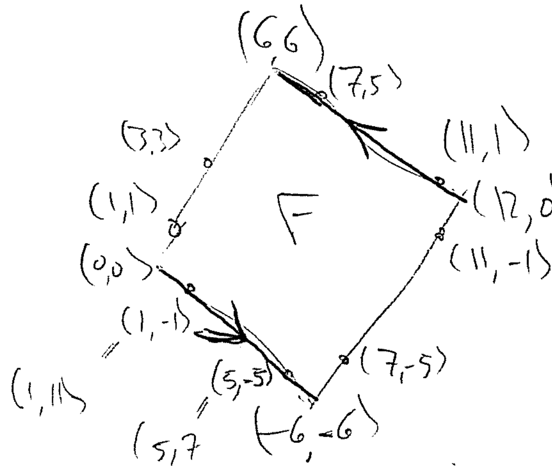
We draw a picture for the case $n=2$.

Claim $(\mathbb{R}/12\mathbb{Z})^2/\mathrm{Sym}(2)$ is the Möbius strip.

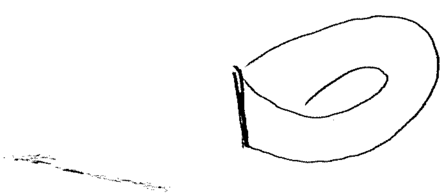
Pr We find a fundamental domain F (i.e. a subset of $(\mathbb{R}/12\mathbb{Z})^2$ whose two translates cover $(\mathbb{R}/12\mathbb{Z})^2$ and only intersect on their boundaries) and then glue the boundary of the fundamental domain according to the action.



Recall that the torus $(\mathbb{R}/12\mathbb{Z})^2$  is the quotient of $\mathbb{R}\times\mathbb{R}$.



Thus, the points of this orbifold are two note chords, and a path in this orbifold is a voice leading.



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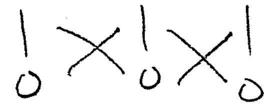
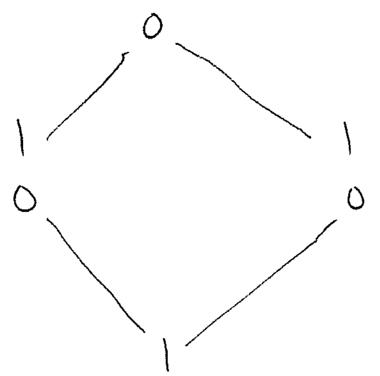
There are similar pictures for $n>2$.

Let's look at some actual voice leadings on this orbifold. Do Movie 1

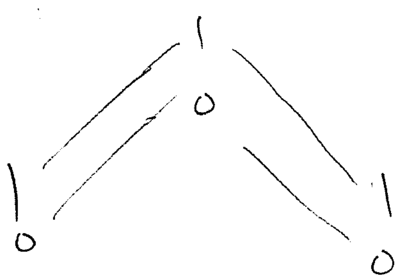
Ex



First look at circle, then look at path on orbifold, point out which sides get identified.

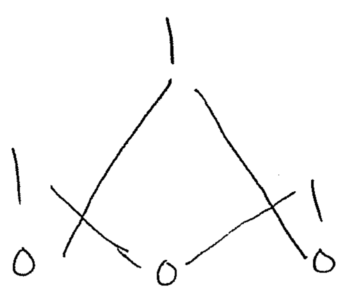


one goes down, the other goes up. (look at numbers)



they both go up, look at numbers.

None of these voice leadings is good.



Don't explain second example of Movie 2.

Do Movie 2.

Do first part of Movie 3, pointing out: no crossovers, voices do not all move in same dir. by same amount, minimal distance.