Selected REU Projects in Mathematical Music Theory

Thomas M. Fiore

http://www-personal.umd.umich.edu/~tmfiore/

January 9, 2011

University of Chicago REU Program

These student projects were completed in the summers of 2006, 2007, 2009 in the University of Chicago REU Program. Features of the REU:

- Senior faculty and post-docs give lecture series, with problems
- Undergraduate participants write papers on topics related to the lectures
- Graduate students and the speakers mentor the students on their papers
- Undergraduates teach in various outreach programs, some also give a talk at the end
- 98 undergraduate participants in Summer 2009, all from UChicago
- NSF Funded
- Organized by Peter May
- Links to projects on my Music webpage.

Project 1: John Sternberg on GIS's, Simply Transitive Group Actions, and Hindemith, 2006

In his paper, John Sternberg recalled

- generalized interval systems and their transposition groups, group actions, the equivalence of group actions with group homomorphisms into symmetric groups.

He proved

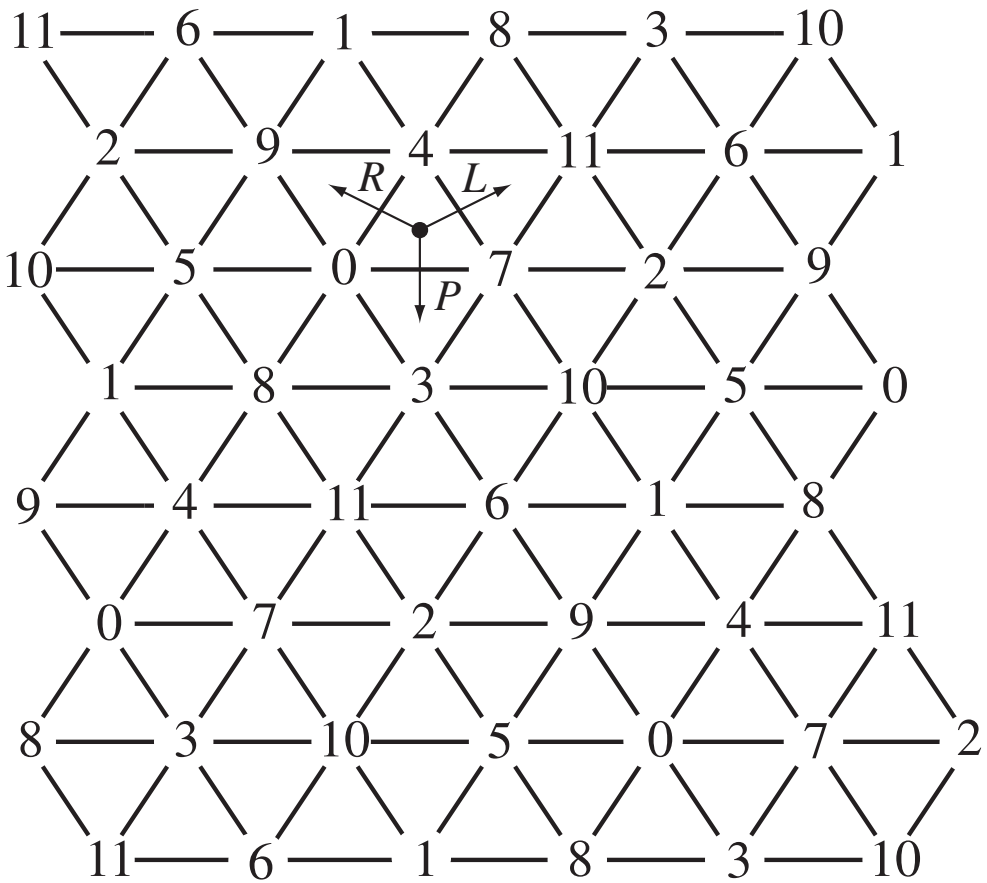
- a generalized interval system gives rise to a simply transitive group action via the transposition group (Lewin).

Then he analyzed

- "Fuga Tertia in F" from Hindemith's *Ludus Tonalis* using T_3 and a contextual inversion operator.

Project 2: Adam Koss on Mazzola's Harmonic Band as a Generalized *Tonnetz*, 2009

Recall the GIS $(\mathbb{Z}_{12}, (\mathbb{Z}_{12}, +), int)$ and the neo-Riemannian *Tonnetz*.



Project 2: Adam Koss on Mazzola's Harmonic Band as a Generalized *Tonnetz*, 2009

Notice any two vertices have a difference of 3, 4, or 7, and the subset $\{3, 4, 7\}$ generates the abelian interval group $(\mathbb{Z}_{12}, +)$. The set of vertices of the neo-Riemannian *Tonnetz* is the musical space of the GIS, and there is an edge (a, b) if and only if $(a, b) \in int^{-1}\{3, 4, 7\}$.

Similarly, given an abelian generalized interval system $(S, IVLS, int)$ and a generating $X \subseteq IVLS$, we obtain a *generalized Tonnetz* with vertex set S and edges $int^{-1}X$. See

Žabka, Marek. Generalized *Tonnetz* and Well-Formed GTS: a Scale Theory Inspired by the Neo-Riemannians. In the *Proceedings of the Second International Conference of the Society for Mathematics and Computation in Music*, Chew, Childs, Chuan, Eds., 2009.

Project 2: Adam Koss on Mazzola's Harmonic Band as a Generalized *Tonnetz*, 2009

In his paper, Adam Koss

- Compared and contrasted the chromatic and diatonic scales
- Determined and drew the vertices and the edges of the neo-Riemannian *Tonnetz* with GIS $(\mathbb{Z}_{12}, (\mathbb{Z}_{12}, +), int)$ with $X = \{3, 4, 7\}$
- Determined and drew the vertices and the edges of Mazzola's Harmonic Band, which arises from the GIS $(\mathbb{Z}_7, (\mathbb{Z}_7, +), int)$ with $X = \{2, 4\}$
- These drawings illustrated the neo-Riemannian *Tonnetz* as a torus and Mazzola's Harmonic Band as a Möbius strip.

Project 3: Padraic Bartlett on the Topos of Triads, 2007

In his project, Padraic Bartlett

- Wrote an exposition of Thomas Noll's work on the Topos of Triads

- Answered the question: to what extent is the major triad characterized by the natural bijection between its set of consonant sub-pairs and its affine stabilizer?
  1. Wrote a computer program to gather data for conjectures
  2. Proved his conjectures.

Project 3: Padraic Bartlett on the Topos of Triads, 2007

See
Noll, Thomas. The Topos of Triads. In *Colloquium on Mathematical Music Theory*. H. Fripertinger, L. Reich (Eds.), Grazer Math. Ber. 347, 2005.

Consider affine maps $\mathbb{Z}_{12} \to \mathbb{Z}_{12}$, these are maps of the form $x \mapsto mx + b$. The *triadic monoid* $\mathcal{T}$ consists of those affine maps which preserve the C-major chord as a set. $|\mathcal{T}| = 8$.

C-major is a *concord* of consonant intervals, i.e., the 8 consonant subpairs correspond precisely to $\mathcal{T}$.

$Sets^{\mathcal{T}}$ = sets equipped with a $\mathcal{T}$-action, their maps.

Noll investigated this *topos* and found an amazing amount of musical structure in it, and used it to analyze a work by Skrjabin.

Other Projects

- Brad Trotter. Word theory and the musical scale, 2009. (Clampitt–Domínguez–Noll)
- Christopher Wood. Abstracting tonality: triads and uniform triadic transformations in an atonal context, 2009. (Julian Hook and Jack Douthett)
- Kenneth Oshita. Hexatonic systems, 2009. (Richard Cohn)
- Hannah Gordon, The neo-Riemannian group.
- Milan Kidd, The neo-Riemannian *Tonnetz* and the Beatles.