Mathematics and Music
2006 REU
Lecture 4 Problems
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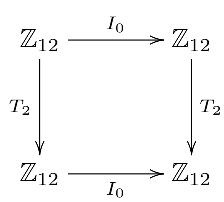
1. Calculate $P\langle 1,5,8\rangle$, $L\langle 10,6,3\rangle$, and $R\langle 9,1,4\rangle$ using the table below.

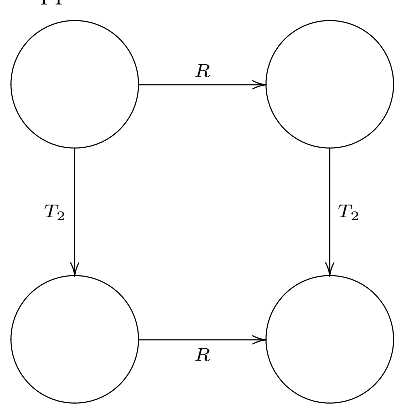
Transposed Forms	Inverted Forms
$C = \langle 0,4,7\rangle$	$\langle 0,8,5\rangle = f$
$C\sharp = D\flat = \langle 1,5,8\rangle$	$\langle 1,9,6\rangle = f\sharp = g\flat$
$D = \langle 2,6,9\rangle$	$\langle 2,10,7\rangle = g$
$D\sharp = E\flat = \langle 3,7,10\rangle$	$\langle 3,11,8\rangle = g\sharp = a\flat$
$E = \langle 4,8,11\rangle$	$\langle 4,0,9\rangle = a$
$F = \langle 5,9,0\rangle$	$\langle 5,1,10\rangle = a\sharp = b\flat$
$F\sharp = G\flat = \langle 6,10,1\rangle$	$\langle 6,2,11\rangle = b$
$G = \langle 7,11,2\rangle$	$\langle 7,3,0\rangle = c$
$G\sharp = A\flat = \langle 8,0,3\rangle$	$\langle 8,4,1\rangle = c\sharp = d\flat$
$A = \langle 9,1,4\rangle$	$\langle 9,5,2\rangle = d$
$A\sharp = B\flat = \langle 10,2,5\rangle$	$\langle 10,6,3\rangle = d\sharp = e\flat$
$B = \langle 11,3,6\rangle$	$\langle 11,7,4\rangle = e$

2. Does the diagram



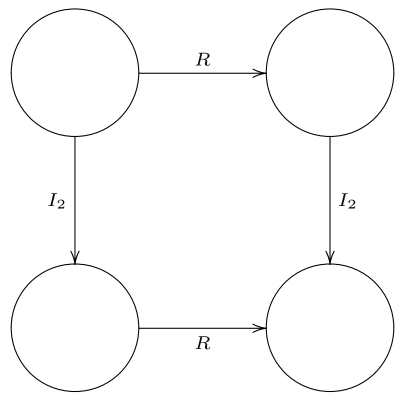
commute? Compare this with Problem 4 of Lecture 3 Problems.

3. Insert $\langle 0,4,7\rangle$ into the upper left circle of the network



and verify that the result is the same no matter which path you take. Compare this with Problem 2 above. The contextual inversion R is in the dual group to the T/I group, but the inversion I_0 is not.

4. Insert $\langle 0, 4, 7\rangle$ into the upper left circle of the network



and verify that the result is the same no matter which path you take. Compare this with Problem 2 above.

5. Are the triangle and the circle qualitatively the same? In other words, can we obtain one from the other by shrinking, stretching, or twisting? How about the triangle and the shape ∞?

6. Calculate each of the following.

$$R\langle 0, 4, 7\rangle$$
$$L \circ R\langle 0, 4, 7\rangle$$
$$R \circ L \circ R\langle 0, 4, 7\rangle$$
$$L \circ R \circ L \circ R\langle 0, 4, 7\rangle$$

Next translate the results into chord names using the table above. How does this relate to the chord progression in the second movement of Beethoven's Ninth Symphony?

7. **Challenge!** Prove that the neo-Riemannian PLR group is dihedral of order 24. (Hint: Continue Problem 6 to get the entire sequence of 24 chords, the first 19 of which appear in Beethoven's 9th Symphony. Conclude from this that the PLR group has at least 24 elements. Then show $R(LR)^3 = P$. Then set $s = LR$ and $t = L$ and show $s^{12} = 1$, $t^2 = 1$, and $tst = s^{-1}$. Lastly, show that the PLR group has 24 elements. They are other ways to prove this. For example, the general theory says that the transposition group of a GIS is anti-isomorphic to its interval group, and dual GIS's have the same interval group. Then the claim follows because PLR and T/I are dual.)