

The slide features a decorative background with a vertical yellow bar on the left, a horizontal green bar at the bottom, and a dark green textured bar on the left side. The text is centered on the green bar.

PSYC 4100

COGNITIVE PSYCHOLOGY

ARLO CLARK-FOOS

Part XIII  
Problem Solving

# EXAMPLES: ANALOGY

- © What word completes the analogy?

Merchant: Sell :: Customer: \_\_\_\_\_

Lawyer: Client :: Doctor: \_\_\_\_\_

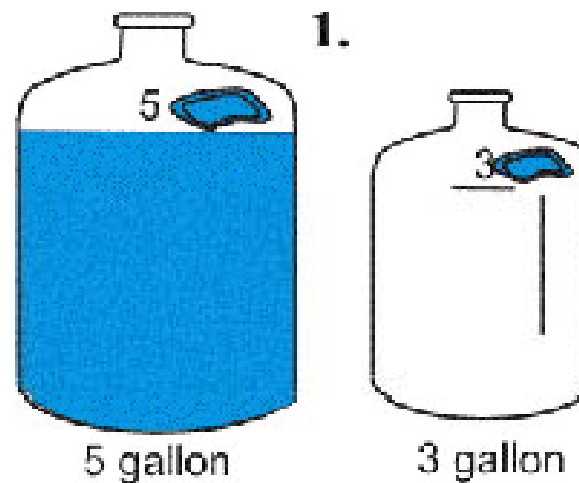


## EXAMPLES: MISSIONARIES & CANNIBALS

- ① Five missionaries and five cannibals who have to cross a river find a boat, but the boat is so small that it can hold no more than three persons. If the missionaries on either bank of the river or in the boat are outnumbered at any time by cannibals, they will be eaten. Find the simplest schedule of crossings that will allow everyone to cross safely. At least one person must be in the boat at each crossing.

# WATER JUG (DIE HARD 3)

- ③ You have a 5-gallon and a 3-gallon jug. How could you obtain exactly 4 gallons of water?



# EXAMPLES: RED ADAIR



# NEWELL & SIMON (1972)

## Four Features of a Problem

- ① A **goal** or something that constitutes a solution to the problem
- ① A description of the **objects** relevant to achieving a solution.
- ① A set of **operations/actions** that will help achieve the solution.
- ① A set of **constraints** that cannot be violated in the course of solving the problem.

*Well-defined vs. Ill-defined problems*



# SPECIFICATION OF PROBLEMS

- ◎ *Well-defined*

- ◎ Subtract 35 from 492

- ◎ Drive to Atlanta with good directions

- ◎ *Ill-defined*

- ◎ Solve world hunger

- ◎ Have an interesting career



# PROBLEM SOLVING

- ◎ Initial and Goal states
- ◎ Different approaches
- ◎ What is the most important aspect of solving a problem?





## REPRESENTATION: PAPER FOLDING

- ◎ Picture a large sheet of paper (of normal thickness: .004 inches).
- ◎ In your imagination, fold it once (now having two layers).
- ◎ Fold it once more (now having four layers), and continue in your mind's eye folding it upon itself fifty times.
- ◎ How thick is this 50-times folded paper?

# PAPER FOLDING: LET'S DO THE MATH

Thickness of a Single Sheet  
of Paper = .004 inches

$$2^{50} = 1,125,899,906,842,624$$



$$\begin{array}{r} 2^{50} \\ \times \underline{.004} \\ \hline 4,503,599,627,370.496 \text{ inches} \end{array}$$

$$\frac{4,503,599,627,370.496}{12} = 375,299,968,947.541 \text{ feet}$$

$$\frac{375,299,968,947.541}{5280} = 71,079,540.57 \text{ miles}$$

**Solve with an Algorithm**

# PAPER FOLDING: A QUICK COMPARISON



240,000 miles



91,000,000 miles



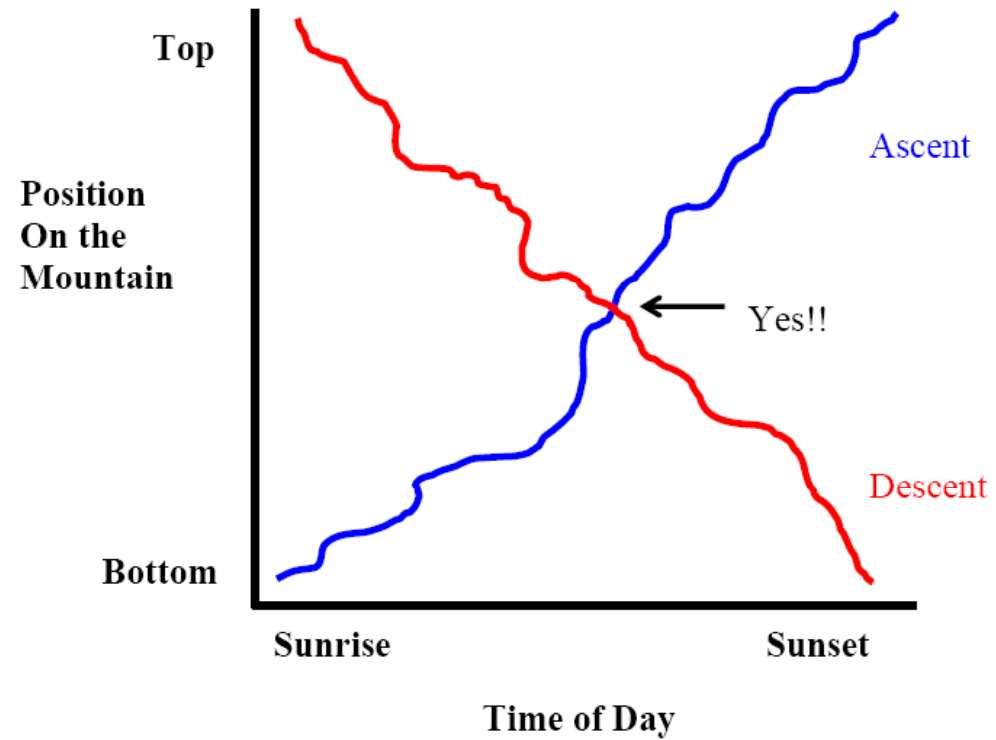
= 71,079,540 miles



# DUNCKER'S MONK PROBLEM

- ⊙ One morning, exactly at sunrise, a Buddhist monk began to climb a tall mountain. A narrow path, no more than a foot or two wide, spiraled around the mountain to a glittering temple at the summit. The monk ascended at varying rates of speed, stopping many times along the way to rest and eat dried fruit he carried with him. He reached the temple shortly before sunset. After several days of fasting and meditation, he began his journey back along the same path, starting at sunrise and again walking at variable speeds with many pauses along the way. His average speed descending was, of course, greater than his average climbing speed.
- ⊙ Prove that there is a spot along the path that the monk will occupy on both trips at precisely the same time of day.

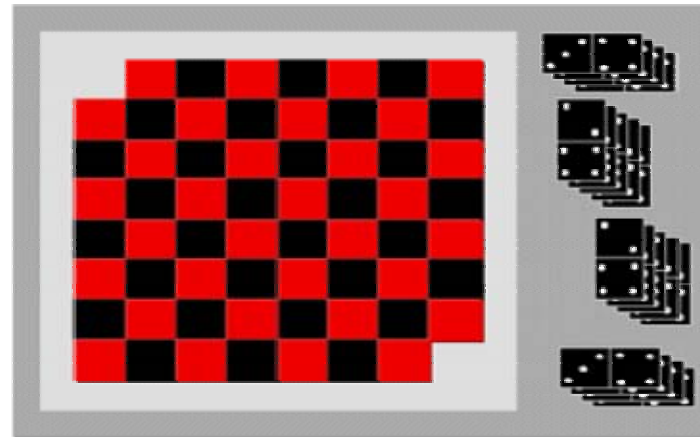
# THE MONK PROBLEM



# MUTILATED CHECKERBOARD

Suppose that a black square has been cut from each corner of a Checkerboard so that it now contains 62 squares. You are given a set of 31 dominoes, where each domino covers exactly 2 squares of the checkerboard. Can you specify an arrangement of dominoes on this mutilated checkerboard?

■ ← missing square



■ ← other missing square



# ALGORITHMS VS. HEURISTICS

- ⊙ Algorithms
  - ⊙ Examples
- ⊙ Heuristics
  - ⊙ Examples
  - ⊙ 4 types

# ALGORITHMS: BUILD A DESK







# PROBLEM SOLVING HEURISTICS

1. Inferences
2. Means-End Analysis/Difference Reduction
3. Working Backwards
4. Analogy



# INFERENCES: TRAIN/BIRD PROBLEM

- ③ Two train stations are 50 miles apart. At 2 PM, two trains start out, one from each station. At the same time, a bird flies in front of the first train, toward the second train. When the bird gets to the second train, it turns back toward the first train. The bird does this until the two trains meet.
- ③ If the trains travel at 25 MPH, the bird flies at 100 MPH, how many miles has the bird flown when the two trains meet?

# TRAIN/BIRD PROBLEM

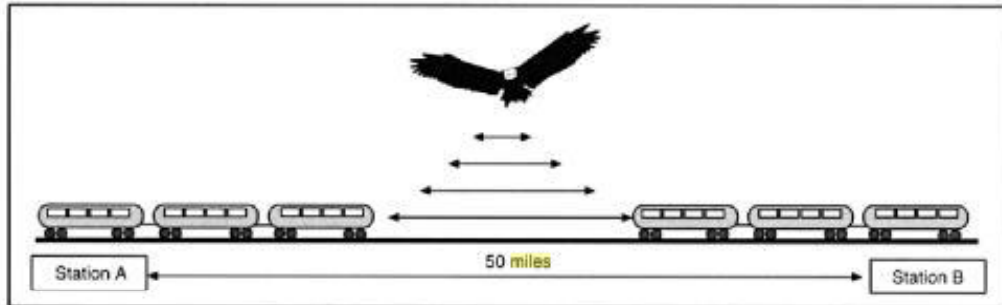


Figure 3.15. The Mad Bird problem from the bird's point of view.

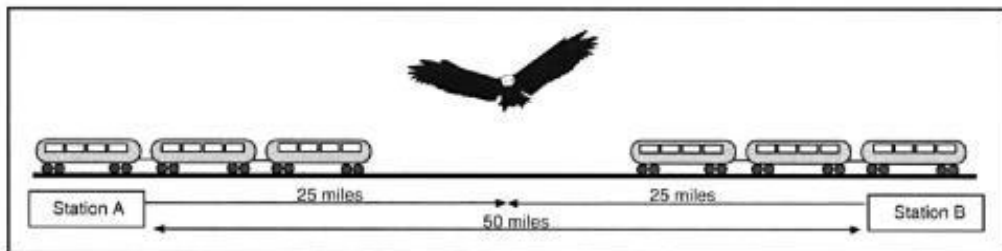
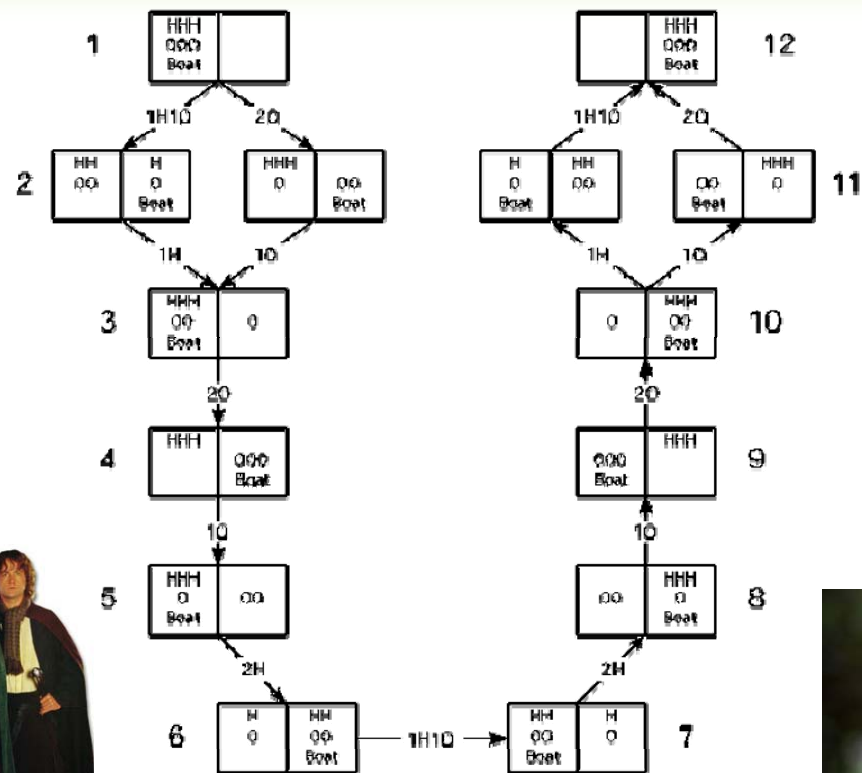


Figure 3.16. The Mad Bird problem from the trains' point of view.

# HOBBIT/ORC PROBLEM





# MEANS-END ANALYSIS: HILL CLIMBING

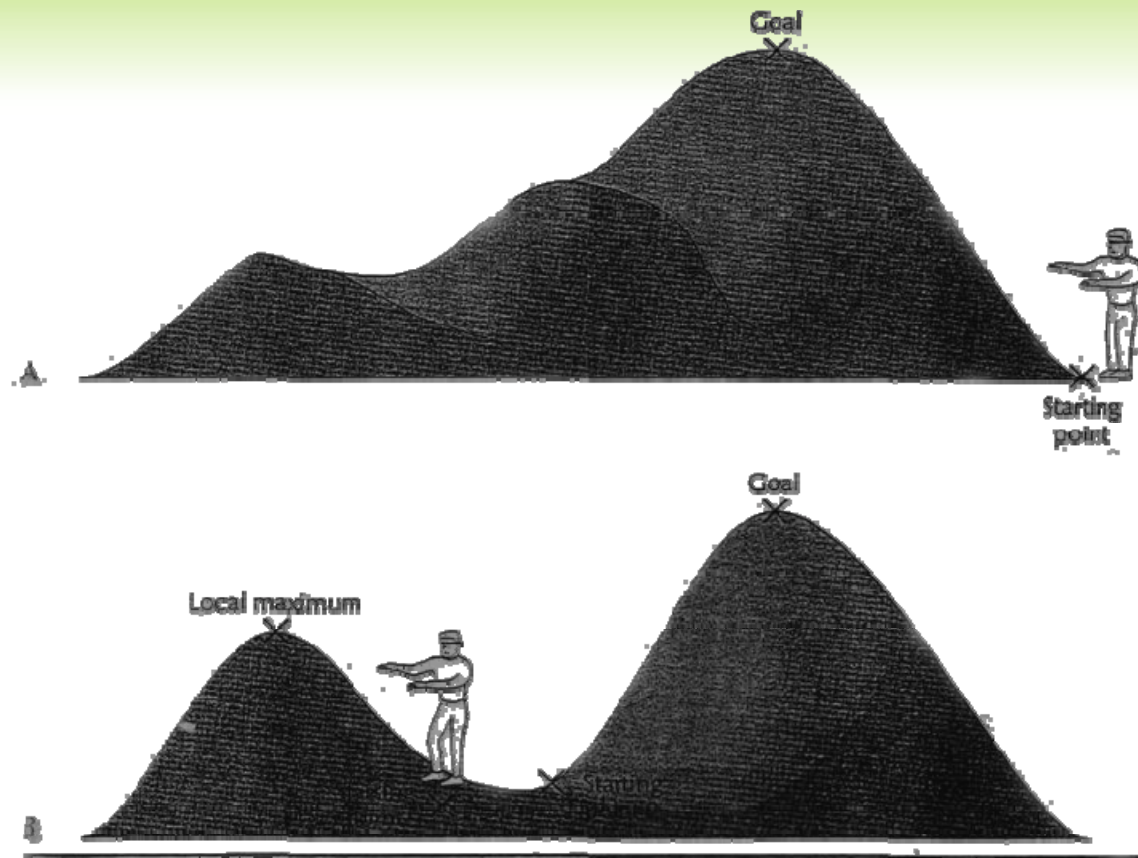
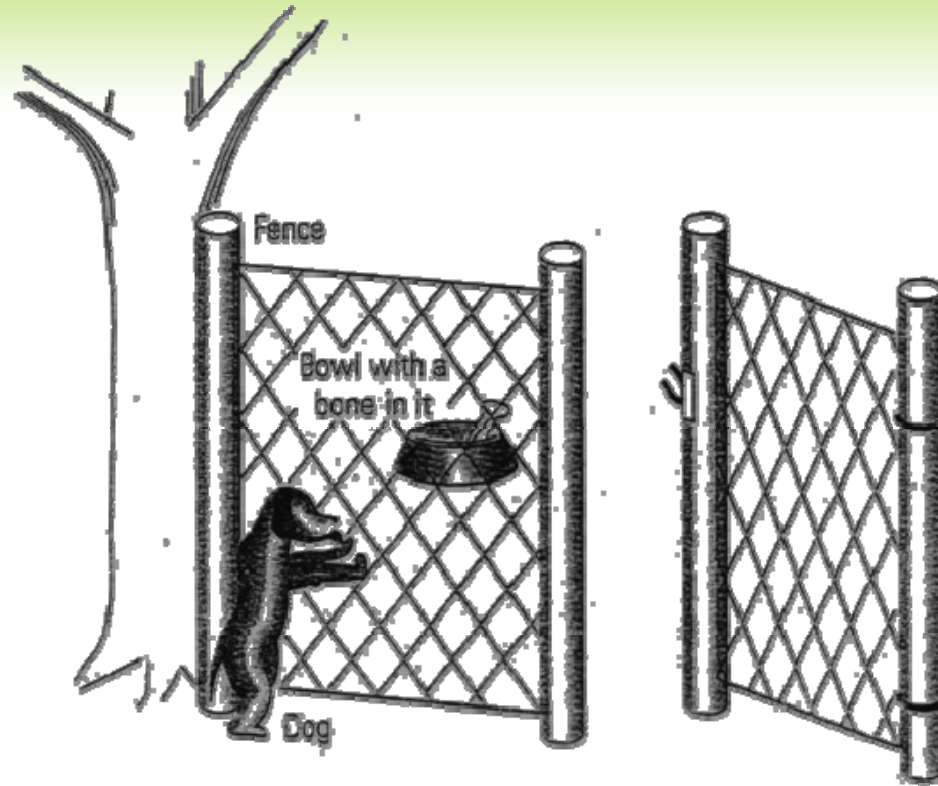


Figure 12.3

**Illustration of Hill-Climbing Heuristic** The terrain represents the closeness to problem solution, with higher elevation being closer to the goal. Panel A shows a blindfolded person moving steadily up the hill by feeling the terrain. Panel B shows the problem of local maximum for this heuristic.

# PROBLEMS WITH MEANS-END ANALYSIS?



According to the hill-climbing strategy, the dog should, at each step, choose a path that moves it closer and closer to the goal. However, this strategy will fail here, since the dog needs first to move away from the bone in order to reach the bone.

# WORKING BACKWARDS: MATH PROBLEMS

- ◎ Mary has some jelly beans. Joan had 3 times as many as Mary but ate 4 and now she has 5. How many jelly beans does Mary have?





# ANALOGY

- ⊙ Radiation Problem
- ⊙ Solutions?
- ⊙ Hint 1. Similar to a problem you heard earlier in the lecture.
- ⊙ Hint 2. Remember Red Adair
- ⊙ Military Problem

# GICK AND HOLYOAK (1983)

⊙ Solve Problem: Radiation [10%]

⊙ Read/Remember Another Problem (Military)

Solve Radiation Problem [30%]

⊙ Read/Remember Another Problem (Military)

*Get Hint: In solving this problem, you may find that one of the stories you read before will give you a hint for solving the problem.*

Solve Radiation Problem [70%]

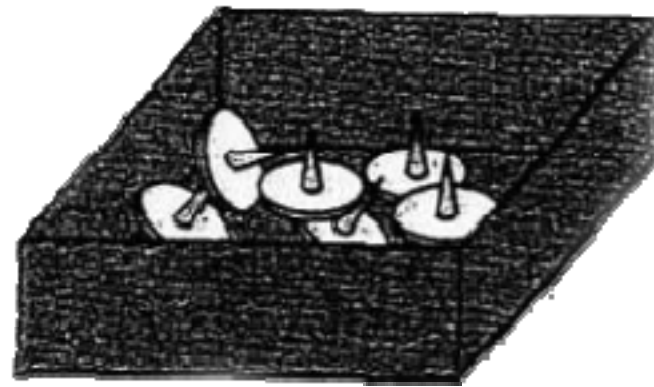
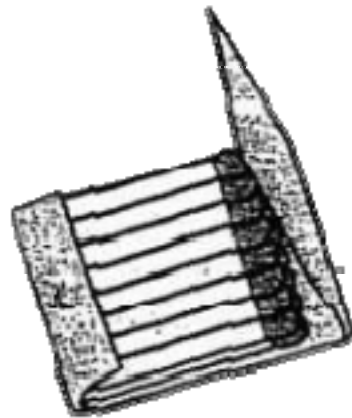


# FUNCTIONAL FIXEDNESS

- ③ Definition
- ③ Duncker and Maier problems
- ③ 2 explanations

# DUNCKER'S PROBLEM

The goal is to place three small candles at eye level on a door. Among other objects on a nearby table are candles, a few tacks, three small boxes about the size of matchboxes, and several matches.



# MAIER'S STRING PROBLEM

Two strings hang from a ceiling but are too far apart to allow a person to hold one and walk to the other. On a nearby table are a book of matches, a screwdriver, and some keys. How could the strings be tied together?





# SET EFFECTS











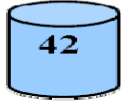




- ③ Definition
- ③ Examples of set effects
- ③ Why is this difficult?

# SERIES COMPLETION

2	4	8	16	32	64	128	___	___
O	T	T	F	F	S	S	___	___

**Set Effects**

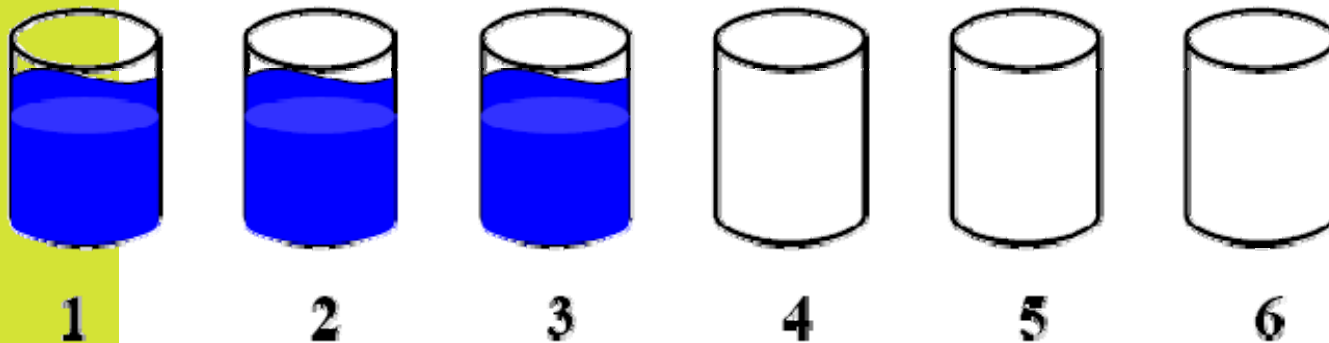
# LUCHIN'S WATER JUG PROBLEM (EINSTELLUNG EFFECT)

	<u>Jug A</u>	<u>Jug B</u>	<u>Jug C</u>	<u>Goal</u>	<u>Formula</u>
1.				100	$B - 2C - A$
2.				99	$B - 2C - A$
3.				5	$B - 2C - A$
4.				21	$B - 2C - A$
5.				31	$B - 2C - A$

*Don't be blind*

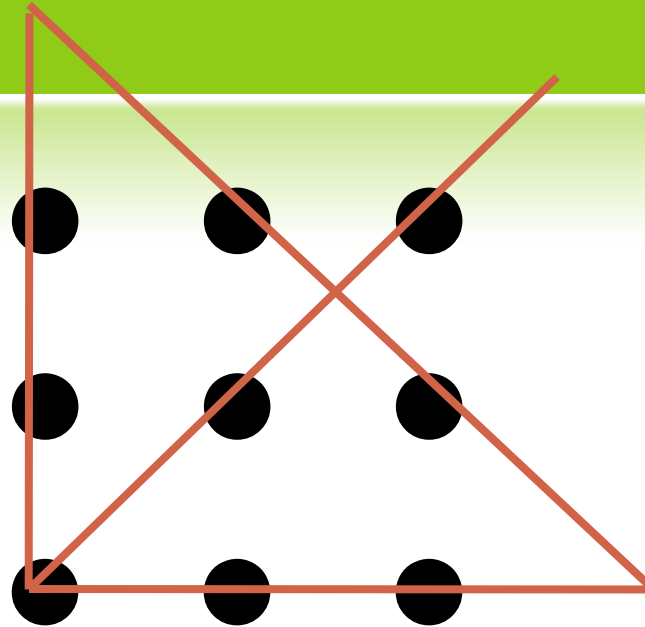


# SET EFFECTS: DRINKING GLASS PROBLEM



Six drinking glasses stand in a row with the first three full of water and the next three empty. By handling and moving only one glass at a time how can you arrange the six glasses so that no full glass stands next to another full glass and no empty glass stands next to another empty glass? What is the minimum number of moves to solve this puzzle?

# SET EFFECTS: 9-DOT PROBLEM

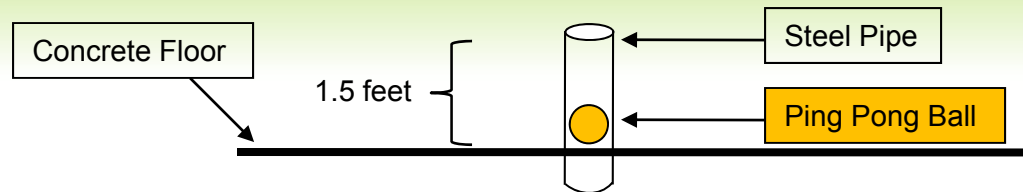


*Think outside the box!*

Without taking your pencil/pen off the paper, and using only four (4) straight lines, touch (or connect) all nine dots

# CULTURAL/TABOO INFLUENCES (VARIANT ON SET EFFECTS)

Adams



- ⊙ A steel pipe is embedded in concrete and the pipe is only 0.5" bigger than the ping pong ball stuck inside.
- ⊙ 6 people in the room (your are one of them)
- ⊙ Objects in the room:
  - 100' of clothes line    A hammer
  - Chisel    Wire coat hanger
  - Box of Wheaties    Light bulb
  - Monkey wrench    A file
- ⊙ List as many ways to get the ping pong ball out of the pipe **WITHOUT** damaging the pipe, ball, or floor.

# INCUBATION



- ◎ Fatigue
- ◎ Unconscious Processing (Silveira, 1971)
  - ◎ “Aha” experience
- ◎ Memory

# CHEAP-NECKLACE PROBLEM

(SILVEIRA, 1971)

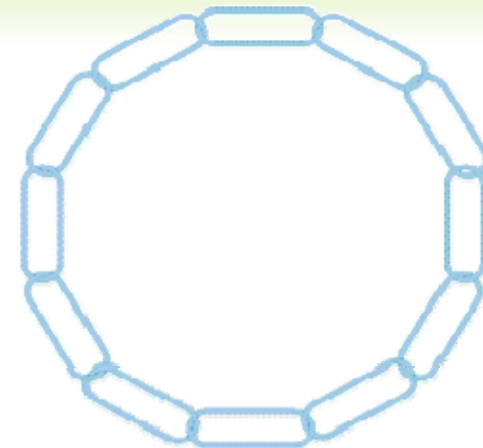
Chain A 

Chain B 

Chain C 

Chain D 

**Given state**

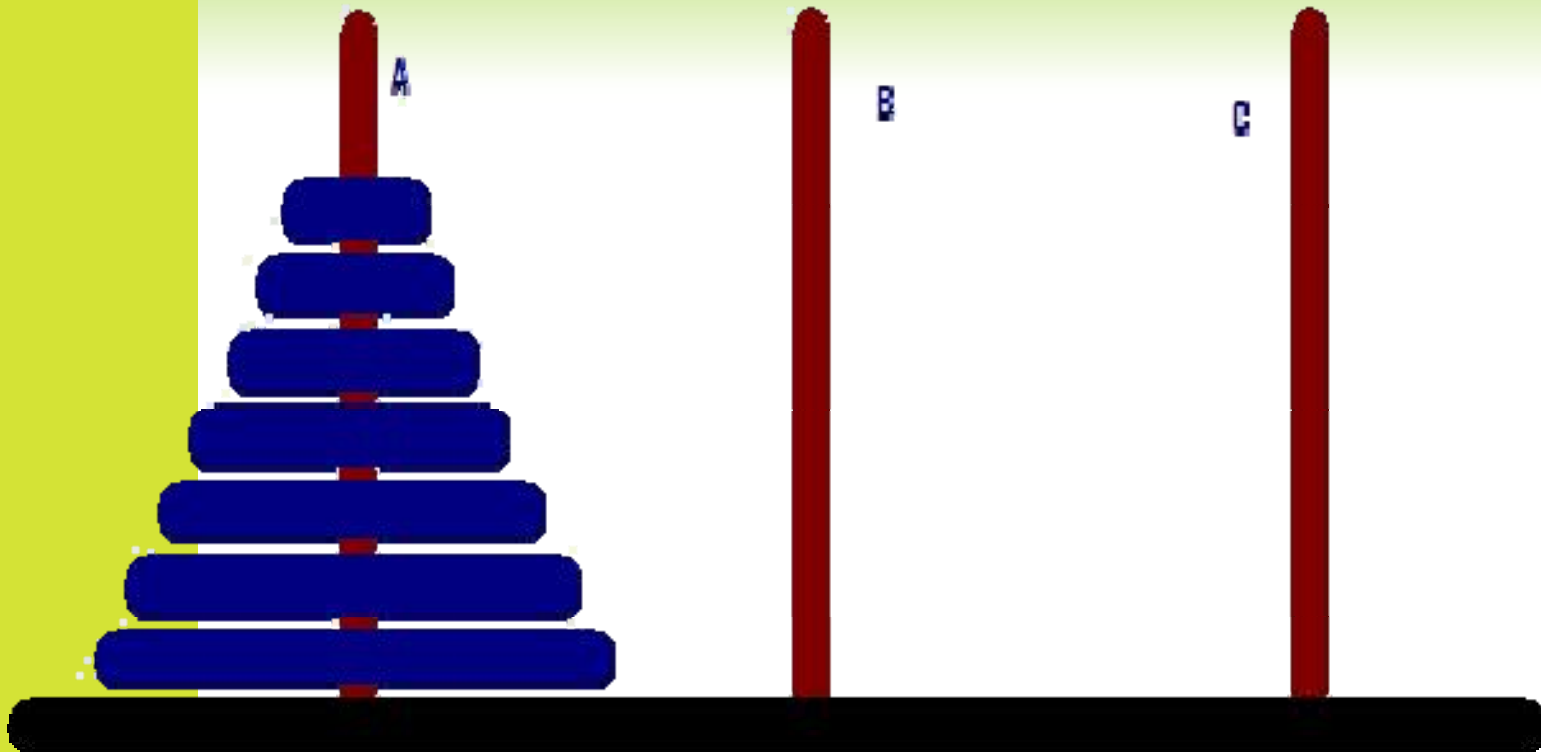


**Goal state**

You are given four separate pieces of chain that are each three links in length. It costs 2¢ to open a link and 3¢ to close one. All links are closed at the beginning of the problem. Your goal is to join all 12 links of chain into a single circle at a cost of no more than 15¢.

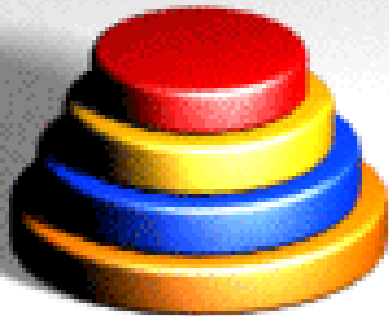
# NON INSIGHT: TOWER OF HANOI

Move the entire stack of blue disks from A to C



- Only one disk may be moved at a time.
- No disk may be placed on top of a smaller disk.

# TOWER OF HANOI SOLUTION



# INSIGHT



Water lilies double in area every 24 hours. At the beginning of summer there is one water lily on the lake. It takes 60 days for the lake to become completely covered with water lilies. On which day is the lake half covered?



# INSIGHT



- A stranger approached a museum curator and offered him an ancient bronze coin. The coin had an authentic appearance and was marked with the date 544 B.C. The curator had happily made acquisitions from suspicious sources before, but this time he promptly called the police and had the stranger arrested.
- Why?

# INSIGHT: RIDDLE

What can run but never walks?

What has a mouth but never talks?

What has a head but never weeps?

What has a bed but never sleeps?





# INSIGHT: RIDDLE

What is greater than God

More evil than the Devil

The poor have it

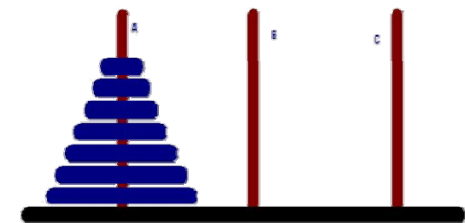
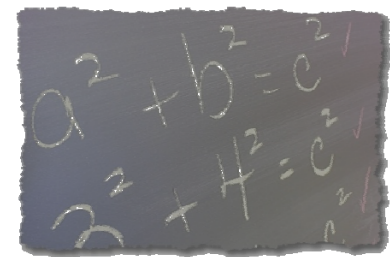
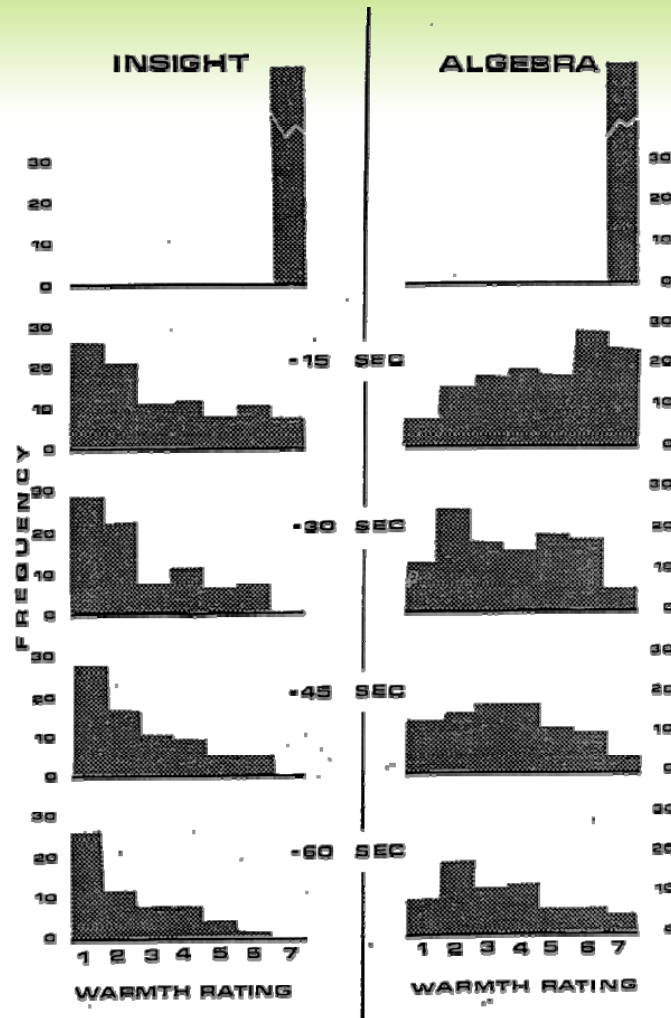
The rich want it

And if you eat it you'll die

# INSIGHT

- ◎ Metcalfe and Wiebe (1987)
  - ◎ Procedure
  - ◎ Warmth ratings
  - ◎ 2 ways to solve a problem

# METCALFE AND WIEBE (1987)



(Metcalfe & Wiebe, 1987)



# PRACTICAL WAYS TO IMPROVE PROBLEM SOLVING

1. Increase your domain knowledge
2. Automate components
3. Follow a systematic plan
4. Draw (warranted) inferences
5. Develop subgoals
6. Work backwards



# PRACTICAL WAYS TO IMPROVE PROBLEM SOLVING

7. Search for contradictions
8. Search for relations among problems
9. Reformulate the representation
10. Represent the problem physically
11. Take a Break when stuck
12. Practice!

# SUMMARY

- ① What is a problem?
- ① Specification of correct attributes in problem solving
- ① Algorithms and heuristics
- ① Difficulties in problem solving