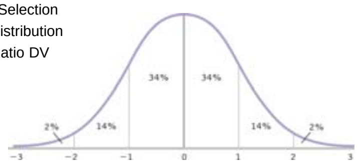
# CHI-SQUARE TESTS


PSYC 381 – STATISTICS
ARLO CLARK-FOOS

## PARAMETRIC TESTS

- **What makes a test parametric?**
  - *Parameters*
    - Estimating the _mean_ of a population(s)
  - What were the assumptions?
    - Random Selection
    - Normal Distribution
    - Interval/Ratio DV

| I. Scale Independent Variable and Scale Dependent Variable | II. Nominal Independent Variable and Scale Dependent Variable |
|---|---|
| Correlation | *z* test |
| Regression | All kinds of *t* tests |
| | All kinds of ANOVAs |




## NONPARAMETRIC TESTS

**When to use**

- **When the dependent variable is nominal or ordinal**
- **Used when the sample size is small**
- **Used when underlying population is not normal (Abby Normal?)**

**Drawbacks**

- **Confidence intervals and effect sizes are harder or impossible**
- **Less statistical power**
  - More likely to commit Type II error
- **Data are less informative (nominal)**

## CHI-SQUARE $\chi^2$

$$X^2 = \sum \left[ \frac{(O-E)^2}{E} \right]$$

- Categorical data may be displayed in contingency tables.
- Two main types of $\chi^2$: Goodness of Fit and Independence
- The chi-square statistic compares the observed frequency in each group to the frequency which would be expected *under the assumption of no association* between the row and column classifications.
- The chi-square statistic may be used to test the hypothesis of no association between two or more groups or populations.
- *Observed* frequencies are compared to *expected* frequencies.

## CHI-SQUARE TEST FOR GOODNESS OF FIT


## CHI-SQUARE ($\chi^2$) TEST FOR GOODNESS OF FIT

IV? DV?

Nope, just one nominal variable and observed rates in each category/level.

New Formula

$$X^2 = \sum \left[ \frac{(O-E)^2}{E} \right]$$

$$df_{\chi^2} = k - 1$$

New Terminology

- Contingency Tables
- Observed Frequency/Count
- Expected Frequency/Count

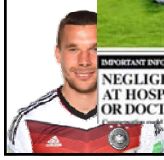
New Symbols

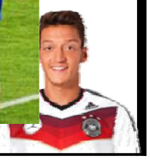
$\chi^2$ E O

## CHI
## GER


As a

- C ?


Lukas Podolski
(b. June 4,1985)


Mesut Özil
(b. Oct 15,1988)

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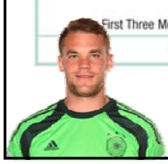
## CHI-SQUARE ($\chi^2$) EXAMPLE: GERMAN SOCCER

$$X^2 = \sum\left[\frac{(O-E)^2}{E}\right]$$

**TABLE 17-3.** Observed Frequencies and Expected Frequencies

The first step in calculating the chi-square statistic is creating two tables, one with cells that display the observed frequencies of birth dates among elite German youth soccer players and one with cells that display the expected frequencies.

| Observed (when elite players were born) | |
|---|---|
| First Three Months of the Year | Last Three Months of the Year |
| 52 | 4 |
| **Expected (based on the general population)** | |
| First Three Months of the Year | Last Three Months of the Year |
| 28 | 28 |


Manuel Neuer
(b. Mar 27,1986)


Jérôme Boateng
(b. Sep 3,1988)

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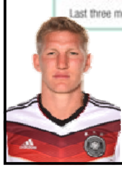
## CHI-SQUARE ($\chi^2$) EXAMPLE: GERMAN SOCCER

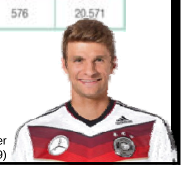
$$X^2 = \sum\left[\frac{(O-E)^2}{E}\right]$$

**TABLE 17-4.** The Chi-Square Calculations

As with many other statistics, we calculate the chi-square statistic using columns to keep track of our work. We calculate the difference between the observed frequency and the expected frequency, square the difference, then divide each square by its appropriate expected frequency. Finally, we add up the numbers in the sixth column to find the chi-square statistic.

| Column 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Category | Observed ($O$) | Expected ($E$) | $O-E$ | $(O-E)^2$ | $\frac{(O-E)^2}{E}$ |
| First three months | 52 | 28 | 24 | 576 | 20.571 |
| Last three months | 4 | 28 | −24 | 576 | 20.571 |


Sebastian Schweinsteiger
(b. Aug 1,1984)


Thomas Müller
(b. Sep 13,1989)

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# CHI-SQUARE ($\chi^2$) EXAMPLE: GERMAN SOCCER

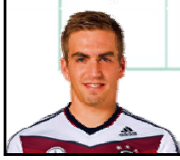
$$X^2 = \sum\left[\frac{(O-E)^2}{E}\right]$$

**Our 'Six Steps' Can Still Apply: Determine cutoff**

**TABLE 17-2.** Excerpt from the $\chi^2$ Table

We use the $\chi^2$ table to determine critical values for a given $p$ level, based on the degrees of freedom.

| | Proportion in Critical Region | | |
|---|---|---|---|
| *df* | 0.10 | 0.05 | 0.01 |
| 1 | 2.706 | 3.841 | 6.635 |
| 2 | 4.605 | 5.992 | 9.211 |
| 3 | 6.252 | 7.815 | 11.345 |



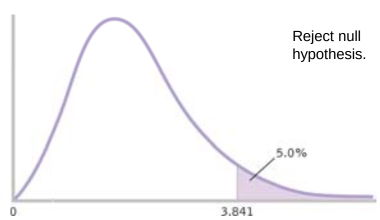
Phillip Lahm
(b. Nov 11,1983)

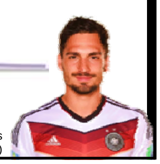
André Schürrle
(b. Nov 6,1990)

# CHI-SQUARE ($\chi^2$) EXAMPLE: GERMAN SOCCER

$$X^2 = \sum\left[\frac{(O-E)^2}{E}\right]$$

**Make a Decision**



Mario Gotze
(b. June 3,1992)

Mats Humels
(b. Dec 16,1988)

# CHI-SQUARE ($\chi^2$) EXAMPLE: GERMAN SOCCER

$$X^2 = \sum\left[\frac{(O-E)^2}{E}\right]$$

**Make a Decision**

More elite German soccer players are born in the first three months of the year compared with the last three months, $X^2(1, N = 56) = 41.142, p < .05$

# CHI-SQUARE ($\chi^2$) TEST FOR INDEPENDENCE

## AN EXAMPLE WITH "THE SIX STEPS"


## CHI-SQUARE ($\chi^2$) TEST FOR INDEPENDENCE

$$X^2 = \sum\left[\frac{(O-E)^2}{E}\right]$$

**Nominal IVs & Non-scalar DV**

**Same Terminology**

- **Contingency Tables**
- **Observed Frequency/Count**
- **Expected Frequency/Count**

**Same Symbols**

- $\chi^2$ E O

**New Formulas**

$$E_{Cell} = \frac{Total_{Column}}{N}(Total_{Row})$$

$$df_{row} = k_{row} - 1$$

$$df_{column} = k_{column} - 1$$

$$df_{X^2} = (df_{row})(df_{column})$$

This is the *reliable* method, used when the situation does not clearly dictate.

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

Patch Adams, M.D.

**Healing power of laughter…**

**= Impregnating power?**

**Researchers are interested in ways of improving pregnancy rates during in vitro fertilization (IVF).**

**(Ryan, 2006)**

**186 women randomly assigned to receive IVF**

- *half before 15 minutes of clown entertainment*

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

**Our data**

**TABLE 17-5.** Observed Pregnancy Rates

This table depicts the cells and their frequencies for the study on whether entertainment by a clown is associated with pregnancy rates among women undergoing in vitro fertilization.

| | Observed | |
|---|---|---|
| | Pregnant | Not Pregnant |
| Clown | 33 | 60 |
| No Clown | 18 | 75 |

Independent Variable (Nominal): Clown or No Clown
Dependent Variable (Not Scalar): Pregnant or Not

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

**Our Six Steps Again**

1. **Identify Populations**
   **Women who receive IVF without seeing a clown**
   **Women who receive IVF before seeing a clown for 15 min**

   **Distribution**
   **One Nominal IVs and Non-Scalar DV so**
   **Chi-Square Test for Goodness of Fit**

   **Assumptions**
   **None, it's Chi-Square!**

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

2. **State Hypotheses**

   Null: There is no association between clown exposure and pregnancy rates in IVF

   Research: There is an association between clown exposure and pregnancy rates in IVF

**TABLE 17-5.** Observed Pregnancy Rates

This table depicts the cells and their frequencies for the study on whether entertainment by a clown is associated with pregnancy rates among women undergoing in vitro fertilization.

| | Observed | |
|---|---|---|
| | Pregnant | Not Pregnant |
| Clown | 33 | 60 |
| No Clown | 18 | 75 |

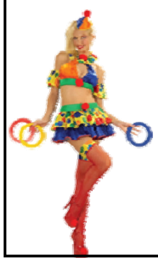
## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

3. Determine Characteristics

**TABLE 17-5.** Observed Pregnancy Rates

This table depicts the cells and their frequencies for the study on whether entertainment by a clown is associated with pregnancy rates among women undergoing in vitro fertilization.

| | Observed | |
|---|---|---|
| | Pregnant | Not Pregnant |
| Clown | 33 | 60 |
| No Clown | 18 | 75 |



$$df_{row} = k_{row} - 1 = 2 - 1 = 1$$

$$df_{column} = k_{column} - 1 = 2 - 1 = 1$$

$$df_{X^2} = (df_{row})(df_{column}) = (1)(1) = 1$$

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

4. Determine Critical Values

**TABLE 17-2.** Excerpt from the $\chi^2$ Table

We use the $\chi^2$ table to determine critical values for a given $p$ level, based on the degrees of free

| | Proportion in Critical Region | | |
|---|---|---|---|
| *df* | 0.10 | 0.05 | 0.0 |
| 1 | 2.706 | 3.841 | 6. |
| 2 | 4.605 | | 9. |
| 3 | 6.252 | 7.815 | 11.3 |
| . | | | |
| . | | | |
| . | | | |

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

5. Calculate Test Statistic

$$X^2 = \sum\left[\frac{(O-E)^2}{E}\right]$$

| | Observed | | |
|---|---|---|---|
| | Pregnant | Not Pregnant | |
| Clown | 33 | 60 | 93 |
| No Clown | 18 | 75 | 93 |
| | 51 | 135 | 186 |

| | Expected | | |
|---|---|---|---|
| | Pregnant | Not Pregnant | |
| Clown | 25.482 | 67.518 | 93 |
| No Clown | 25.482 | 67.518 | 93 |
| | 51 | 135 | 186 |

*Wait, how the heck did we get this...?*

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

$$E_{Cell} = \frac{Total_{Column}}{N}(Total_{Row})$$

**Calculating Expected Frequencies**

| Observed | Pregnant | Not Pregnant | |
|---|---|---|---|
| Clown | 33 | 60 | 93 |
| No Clown | 18 | 75 | 93 |
| | 51 | 135 | 186 |

$$E_{Pregnant,Clown} = \frac{51}{186}(93) = 25.482 \qquad E_{Not\ Pregnant,Clown} = \frac{135}{186}(93) = 67.518$$

$$E_{Pregnant,No\ Clown} = \frac{51}{186}(93) = 25.482 \qquad E_{Not\ Pregnant,No\ Clown} = \frac{135}{186}(93) = 67.518$$

| Expected | Pregnant | Not Pregnant | |
|---|---|---|---|
| Clown | 25.482 | 67.518 | 93 |
| No Clown | 25.482 | 67.518 | 93 |
| | 51 | 135 | 186 |

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

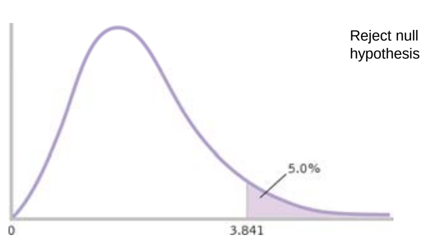
5. **Calculate Test Statistic**

| Category | Observed ($O$) | Expected ($E$) |
|---|---|---|
| Clown; pregnant | 33 | 25.482 |
| Clown; not pregnant | 60 | 67.518 |
| No clown; pregnant | 18 | 25.482 |
| No clown; not pregnant | 75 | 67.518 |

$$X^2 = \sum\left[\frac{(O-E)^2}{E}\right] =$$

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

6. **Make a decision**



## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

**6. Make a decision**

There is an association between exposure to clowns (humor) and pregnancy rates in women who received IVF, $\chi^2$ (1, N = 186) = 6.08, $p$ < .05


## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

**Pop Quiz Hot Shot: How do you display these data?**

**Answer: We graph proportions/percentages rather than frequencies. Otherwise we use a bar graph.**

**TABLE 17-6.** Observed Frequencies with Totals

This table includes the observed frequencies for each of the four cells, along with row totals (93, 93), column totals (51, 135), and the grand total for the whole table (186).

| | Observed | | |
|---|---|---|---|
| | Pregnant | Not Pregnant | |
| Clown | 33 | 60 | 93 |
| No Clown | 18 | 75 | 93 |
| | 51 | 135 | 186 |

**TABLE 17-10.** Conditional Proportions

To construct a graph depicting the results of a chi-square test for independence, we first calculate conditional proportions. For example, we calculate the proportions of women who got pregnant, conditional on having been entertained by a clown post-IVF: 33/93 = 0.355.

| | Conditional Proportions | | |
|---|---|---|---|
| | Pregnant | Not Pregnant | |
| Clown | 0.355 | 0.645 | 1.00 |
| No Clown | 0.194 | 0.806 | 1.00 |

## CHI-SQUARE ($\chi^2$) EXAMPLE: CLOWNING AROUND

**There is an association between exposure to clowns (humor) and pregnancy rates in women who received IVF, $\chi^2$ (1, N = 186) = 6.08, $p$ < .05**

**Can we determine a measure of effect size? Yes!**

$$Cramer's\ V = \sqrt{\frac{\chi^2}{(N)(df_{Row/Column})}} \qquad \sqrt{\frac{6.08}{(186)(1)}} = \sqrt{0.033} = 0.181$$

**TABLE 17-9.** Conventions for Determining Effect Size Based on Cramer's V

Jacob Cohen (1992) developed guidelines to determine whether particular effect sizes should be considered small, medium, or large. The effect size guidelines vary depending on the size of the contingency table. There are different guidelines based on whether the smaller of the two degrees of freedom (row or column) is 1, 2, or 3.

| Effect Size | When $df_{row/column} = 1$ | When $df_{row/column} = 2$ | When $df_{row/column} = 3$ |
|---|---|---|---|
| Small | 0.10 | 0.07 | 0.06 |
| Medium | 0.30 | 0.21 | 0.17 |
| Large | 0.50 | 0.35 | 0.29 |

# EFFECT SIZE

## EFFECT SIZE REVIEW

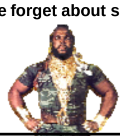
We've one of these already for several different test statistics…

$r^2$ *and* $R^2$ for

Correlation

ANOVA

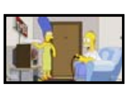
Did we forget about someone?

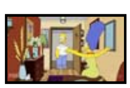


## $R^2$ FOR INDEPENDENT SAMPLES

$$r^2 = \frac{t^2}{t^2 + df}$$

Let's recall when you would use an independent samples *t* test





Effect Size: We can use $r^2$ again

$df_{Total} = 7$

$$t = \frac{(82.25 - 82.6)}{11.847} = -.03$$

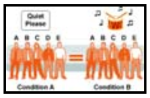
$$r^2 = \frac{t^2}{t^2 + df} = \frac{.0009}{.0009 + 7} = .000128$$

$r^2 = .01$, small effect
$r^2 = .09$, medium effect
$r^2 = .25$, large effect

## $R^2$ FOR DEPENDENT SAMPLES

$$r^2 = \frac{t^2}{t^2 + df}$$

Let's recall when you would use a dependent samples *t* test



Effect Size: We can use $r^2$ again

$df = 4$

$$r^2 = \frac{t^2}{t^2 + df} = \frac{32.7184}{36.7184} = .8911$$

$$t = \frac{(M - \mu_M)}{s_M} = \frac{(-11 - 0)}{1.923} = -5.720$$

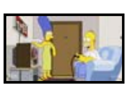
$r^2 = .01$, small effect
$r^2 = .09$, medium effect
$r^2 = .25$, large effect

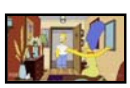
OH THAT COHEN, AGAIN…

# COHEN'S D

## COHEN'S D FOR INDEPENDENT SAMPLES

$$Cohen's\ d = \frac{X_1 - X_2}{\sqrt{s_p^2}}$$





Effect Size: Cohen's *d*

| Women (X) | Men (Y) |
|---|---|
| 84 | 88 |
| 97 | 90 |
| 58 | 52 |
| 90 | 97 |
| $M_X = 82.25$ | 86 |
| | $M_Y = 82.6$ |

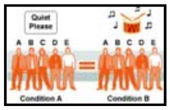
$$s^2_{Pooled} = 124.107 + 187.771 = 311.878$$

$$d = \frac{82.25 - 82.6}{\sqrt{311.878}} = .01982$$

*Rule of Thumb*
*d* = 0.2, small effect
*d* = 0.5, medium effect
*d* = 0.8, large effect

# COHEN'S D FOR DEPENDENT SAMPLES

$$Cohen's\ d = \frac{\bar{X}_D}{s_D}$$




**Effect Size: Cohen's *d***

| Difference Score X-Y | *Deviation Score* (*Score - Mean*) | Squared Deviation (*Score - Mean*)$^2$ |
|---|---|---|
| -11 | 0 | 0 |
| -15 | -4 | 16 |
| -14 | -3 | 9 |
| -4 | 7 | 49 |
| -11 | 0 | 0 |
| *M* = -11 | | $SS_X = 74$ |

$$s = \sqrt{\frac{\Sigma(X-M)^2}{N-1}} = \sqrt{\frac{74}{5-1}} = 4.301$$

$$d = \frac{-11}{4.301} = 2.558$$

*Rule of Thumb*
*d* = 0.2, small effect
*d* = 0.5, medium effect
*d* = 0.8, large effect

# SUMMARY

**Nonparametric vs. Parametric Tests**

**Chi-Square Test**

**Effect Size for *t* Tests**

- $r^2$
- *Cohen's d*

# END OF THE ROAD

**Next**

1. **Exam Review**
2. **Final Exam: ANOVAs, Post-hoc Tests, Chi-Square, Effect Size**
3. ***…?***