To Infinity, and Beyond!

1) Start with three Skittles® (Taste the Rainbow©) of different colors. You are going to pick two Skittles to eat; the order you eat these two in is unimportant.

a) How many different combinations of two Skittles can you make?

b) How many different combinations of three Skittles can you make?

c) Add the numbers you found in a) and b) to the total number of Skittles. Is your answer bigger or smaller than three?

2) Now start with four Skittles, all of different colors. You are going to pick a certain number of them to eat; again, the order you eat them in is unimportant.

a) How many different combinations of two Skittles can you make?

b) How many different combinations of three Skittles can you make?

c) How many different combinations of four Skittles can you make?

d) Add the numbers you found in a), b), and c) to the total number of Skittles. Is your answer bigger or smaller than four?

3) Finally, try starting with five(!) Skittles. You are going to pick a certain number of them to eat; again, the order you eat them in is unimportant.

a) How many different combinations of two Skittles can you make?

b) How many different combinations of three Skittles can you make?

c) How many different combinations of four Skittles can you make?

d) How many different combinations of five Skittles can you make?

e) Add the numbers you found in a), b), c), and d) to the total number of Skittles. Is your answer bigger or smaller than five?

4) a) Write down the numbers you got at the end for problems 1), 2), and 3) again.

b) Can you guess what the final answer for 6 Skittles would be? How about 7?

c) If you couldn't guess, add one to all of your answers from part a). Does that help?

d) Guess a formula for the total number of different combinations of n Skittles. Is it bigger or smaller than n?

5) Now let's look at *infinite* sets. Probably the easiest set you can think of is the counting numbers 1,2,3,4,. . . .

a) How many different combinations of two counting numbers can you make?

b) How many different combinations of three counting numbers can you make?

6) You should have found infinite sets for your answers in 5). The question is: how can you tell whether one infinite set is “bigger” or “smaller” than another?

The notion that we’ll need is an association between two infinite sets called a *function*. The function should associate to each number in the set exactly one number in the other set, and it should not repeat outputs (this is called *one-to-one*). If the sets are the same size, this function should also not miss any numbers in the set it is going to (called *onto*). We say that two infinite sets have the same size if there is a one-to-one, onto function from one set to the other.

For the counting numbers 1,2,3,4,..., take a number and add one to it.

a) What is $f(1)$?

b) What is $f(2)$?

c) What is $f(n)$ for any counting number n?

d) Convince yourself that this function is one-to-one and onto from 1,2,3,4,... to 2,3,4,.... This means that the counting numbers are the same “size” as the counting numbers starting from 2, which is weird because in another sense, one is definitely bigger than the other!

7) a) Can you think of a way to show that all combinations of two counting numbers are really the same “size” as the counting numbers themselves?

b) Think of all the many ways you can make combinations of the counting numbers (finite or infinite combinations). Do you think all such combinations are a bigger infinity than just the counting numbers? Why or why not?

8) The process of taking all combinations of a set is called the *power set.* Here is the most important facts about power sets: even for infinite sets, they are always bigger in size than the set itself!

a) Can you think about how you could use this fact to create an infinity of different infinities?