Prime Time

1) Recall that a *prime number* is a whole number greater than one that is only evenly divisible by one and itself.

a) Find all the prime numbers up to 50.

b) Why do you think the number one is not considered a prime number?

2) You may (or may not) know that every whole number greater than one is either prime or can be expressed as a product of primes. This is called the *prime decomposition* or a whole number.

a) Find the prime decomposition of 101.

b) Find the prime decomposition of 576.

c) Find the prime decomposition of 9999.

3) We're going to run with that last example for a little.

a) Check that the prime decomposition of 999999 is $3^3 \times 7 \times 11 \times 13 \times 37$. Note that 7 is a divisor.

b) Check that the prime decomposition of 9999999999 is $3^2 \times 11 \times 41 \times 271 \times 9091$. Note that 11 is a divisor.

c) Find a prime number *other than 3* that divides 999999999999.

d) Guess how many 9's you'd need to string in a row to ensure divisibility by 19.

e) How many 9's would you need to string in a row to ensure divisibility by 23?

f) How many 9's would you need to string in a row to ensure divisibility by p for any prime $p > 5$?

4) Now we're going to see if the intuition from the previous problem stacks up. Pick a prime p and choose a whole number n that is not divisible by p. I'll choose $p = 3$ and $n = 14$. Then

$$n^{p-1} - 1 = 14^{3-1} - 1 = 195$$

is divisible by $p = 3$. You can see this immediately for the choices I made. This is called *Fermat's Little Theorem* (not to be confused with Fermat's Last Theorem).

a) Try this with your own (relatively small) choices of n and p.

b) What choice of n should you make to get the 9's result from the previous page?

c) For the 9's result, why is it important that we not choose $p = 2$ or $p = 5$?

5) Given a prime p greater than 5, what is the *smallest* number of 9's we need for p to divide a string of that many 9's?

6) Fermat's Theorem has a generalization to any two whole numbers that are *relatively prime*; that is, their greatest common divisor is equal to one. Here's an idea of what happens with 6 and 35:

$35^1 - 1 = 34$ (not divisible by 6)

$35^2 - 1 = 1224$ (divisible by 6)

Try this kind of trick with a few relatively prime numbers and see if you can guess what happens.