Math 412/512 Assignment 3

Due Wednesday, February 2

1) (See Chapter 5, Section E) a) Prove that $\mathbb{Z}_3 \times \mathbb{Z}_2$ is a cyclic group.

b) Prove that $\mathbb{Z}_2 \times \mathbb{Z}_4$ is not a cyclic group.

c) Is $\mathbb{Z}_{30} \times \mathbb{Z}_{77}$ a cyclic group? Either prove or give a counterexample.

d) Conjecture when $\mathbb{Z}_n \times \mathbb{Z}_m$ is cyclic. Don't prove your guess, though- unless you want some extra credit!

2) An *automorphism* is an isomorphism from a group $\langle G, \cdot \rangle$ to itself. The collection of all automorphisms of $\langle G, \cdot \rangle$ is denoted by $Aut(G)$.

a) Prove that $Aut(G)$ is a subgroup of the group of all bijections on G with the operation of function composition.

b) $Aut(\mathbb{Z}_5)$ and $Aut(\mathbb{Z}_6)$ are each isomorphic to $\mathbb{Z}_n$ for some n. Determine the values of n.

3) Determine the center of the group of invertible 2×2 matrices with entries in $\mathbb{R}$ under the operation of matrix multiplication.

4) (See Chapter 8, Section H) a) Show that S_3 is generated by the set $\{(12), (13)\}$.

b) Show that S_n is generated by the set $\{(12), (13), \ldots, (1n)\}$. *Hint:* you may use the fact that S_n is generated by transpositions.

5) (Extra credit) Prove that if $n \geq 4$, the number of elements in S_n which are the product of disjoint transpositions is $\dfrac{n(n-1)(n-2)(n-3)}{8}$.