Math 412/512 Assignment 3

Due Thursday, October 11

1) (#3, Chapter 4) a) List the elements of the subgroups $\langle 20 \rangle$ and $\langle 10 \rangle$ in $\mathbb{Z}_{30}$.

b) Let a be a generator of $\mathbb{Z}_{30}$. List the elements of the subgroup $\langle a^{20} \rangle$ and $\langle a^{30} \rangle$.

2) (#34, Supplementary Exercises for Chapters 1-4) Suppose that G is a group that has exactly one nontrivial proper subgroup. Prove that G is cyclic and $|G| = p^2$ where p is prime.

3) a) Prove that $\mathbb{Z}_2 \oplus \mathbb{Z}_5$ is a cyclic group.

b) Prove that $\mathbb{Z}_2 \oplus \mathbb{Z}_6$ is not a cyclic group.

c) Is $\mathbb{Z}_{49} \oplus \mathbb{Z}_{132}$ a cyclic group? Either prove or give a counterexample.

d) Conjecture when $\mathbb{Z}_n \oplus \mathbb{Z}_m$ is cyclic. Don't prove your guess, though- unless you want some extra credit!

4) We know from class that S_n is not cyclic as soon as $n \geq 3$, hence not generated by a single element. This problem explores subsets that DO generate S_n.

a) Show that S_3 is generated by the set $\{(12), (13)\}$.

b) Show that S_n is generated by the set $\{(12), (13), \ldots, (1n)\}$. *Hint:* you may use the fact that S_n is generated by transpositions.