Math 412/512 Assignment 4

Due Thursday, October 20

1) a) If $\sigma \in S_9$ is given in array notation by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 7 & 9 & 4 & 2 & 8 & 1 & 6 & 3 \end{pmatrix}$$

express σ in cycle notation.

b) If $\psi, \phi \in S_9$, $\psi = (194)(2178)(35)$ and $\phi = (387)(26)(19)$, express $\phi \circ \psi$ and $\psi \circ \phi$ in cycle notation. Calculate $|\phi \circ \psi|$.

c) Check whether any of σ, ψ, or ϕ (as defined in parts a) and b)) are elements of A_9.

2) If $g \in G$, define the *conjugacy class* $\mathrm{Cl}(g)$ of g to be the subset of G consisting of all elements of the form hgh^{-1} for $h \in G$. In symbols,

$$\mathrm{Cl}(g) = \{k \in G | k = hgh^{-1} \text{ for some } h \in G\}.$$

a) If $\sigma = (12) \in S_3$, determine $\mathrm{Cl}(\sigma)$.

b) If $g \in \mathcal{Z}(G)$, determine $\mathrm{Cl}(g)$.

c) For which $g \in G$ do we have $\mathrm{Cl}(g) \leq G$? Prove that your answer is correct.

3) (#46, Chapter 5) Let $H \leq S_n$ and suppose the order of H is odd. Show that $H \leq A_n$.

4) Recall that D_n is the group of rotations and reflections that preserve a regular n-gon.

a) Show that S_3 is isomorphic to D_3.

b) The proof of Cayley's Theorem given in class yields a subgroup of S_8 that is isomorphic to D_4. After labeling the elements of D_4, explicitly produce the subgroup given by the proof, i.e., determine all the elements.

5) (#10, Chapter 6) Let G be a group. Prove that the mapping $\alpha : G \to G$, $\alpha(g) = g^{-1}$ for all $g \in G$, is an automorphism if and only if G is abelian.