Dihedral Groups

(Section 2.3)

Recall: (symmetries of the square and equilateral triangle)

We denoted by D_3 the symmetries of the equilateral triangle, and by D_4 the symmetries of the square.

We will extend this notation to **all** regular polygons!

Definition : (finite dihedral groups) Let P be a regular polygon. We denote by D_n the group of all symmetries of P. D_n is the dihedral group of order $2n$.

Q: Why $2n$ instead of n?

A: An n-sided regular polygon has rotational symmetries e (do nothing) and powers of rotation by $\frac{2\pi}{n}$ radians, denoted $R_{\frac{2\pi}{n}}$.

Then there are n flips about either axes of symmetry through vertices, through sides, or through a vertex and a side.

Total of $2n$ symmetries!