2/18/11

New Problems

1) Let $G = [0, 1)$ with operation

$$x \dotplus y = \begin{cases} x+y, & \text{if } x+y < 1 \\ x+y-1, & \text{if } x+y \geq 1 \end{cases}$$

Show that this is a group. Is it abelian group?

2) Let $G = \langle \mathbb{R}, + \rangle$ and $H = \{x \in \mathbb{R} : x^2 \in \mathbb{Q}\}$

Is H a subgroup of G?

3) $U(n) = \{m \in \mathbb{Z}_n : \exists\, k \in \mathbb{Z}_n$ with $mk = 1 \pmod n\}$

Show that $|U(n)| = \varphi(n)$
= the number of positive integers less than and relatively prime to n

3 from last time

For $x \in \mathbb{R}$ define $\varphi_x(f) = f(x)$

Let $f, g \in C(\mathbb{R})$

WTS: $\varphi_x(f+g) = \varphi_x(f) + \varphi_x(g)$

$$\begin{aligned} \varphi_x(f+g) &= (f+g)(x) \\ &= f(x) + g(x) \\ &= \varphi_x(f) + \varphi_x(g) \end{aligned}$$

$\ker \varphi = \{f \in C(\mathbb{R}) \mid \varphi_x(f) = 0\}$

$\Rightarrow \ker \varphi = \{f \mid f(x) = 0\}$

1) Closure is assumed

Identity : 0

$\forall x \in G \quad x + 0 = x$

$0 + x = x$

Inverse: $\forall x \in G \quad x \neq 0$

$(1 - x) = x^{-1}$

$x + (1 - x) = 1$

Since $1 \geq 1$, $1 - 1 = 0$

If $x = 0$ the inverse is 0

$0 + 0 = 0$

yes - abelian group.

2) identity $= 0$ ✓

inverse $= -x$ ✓

closure = doesn't work

counter ex: $x = 1$, $y = \sqrt{2}$

$(x + y)^2 = 3 + 2\sqrt{2} \notin \mathbb{Q}$

$\therefore$ not closed and not a subgroup.