Multiplication Tables

(Section 1.3)

Monty Hall goat question:

There are 3 closed doors.

Behind 2 of them, there are goats. Behind the remaining door, there is a shiny new car.

You choose door 1, say.

Before door 1 is opened, Monty Hall says that you can have whatever is behind the door you choose, but you can only open one door. He opens a door that you did not choose and there is a goat behind it. He then offers you the opportunity to switch your choice. Strategically, should you switch or should you stay?

Make a table

Door 1	Door 2	Door 3	Stay outcome	Switch outcome
goat	goat	car	lose	win
goat	car	goat	lose	win
car	goat	goat	win	lose

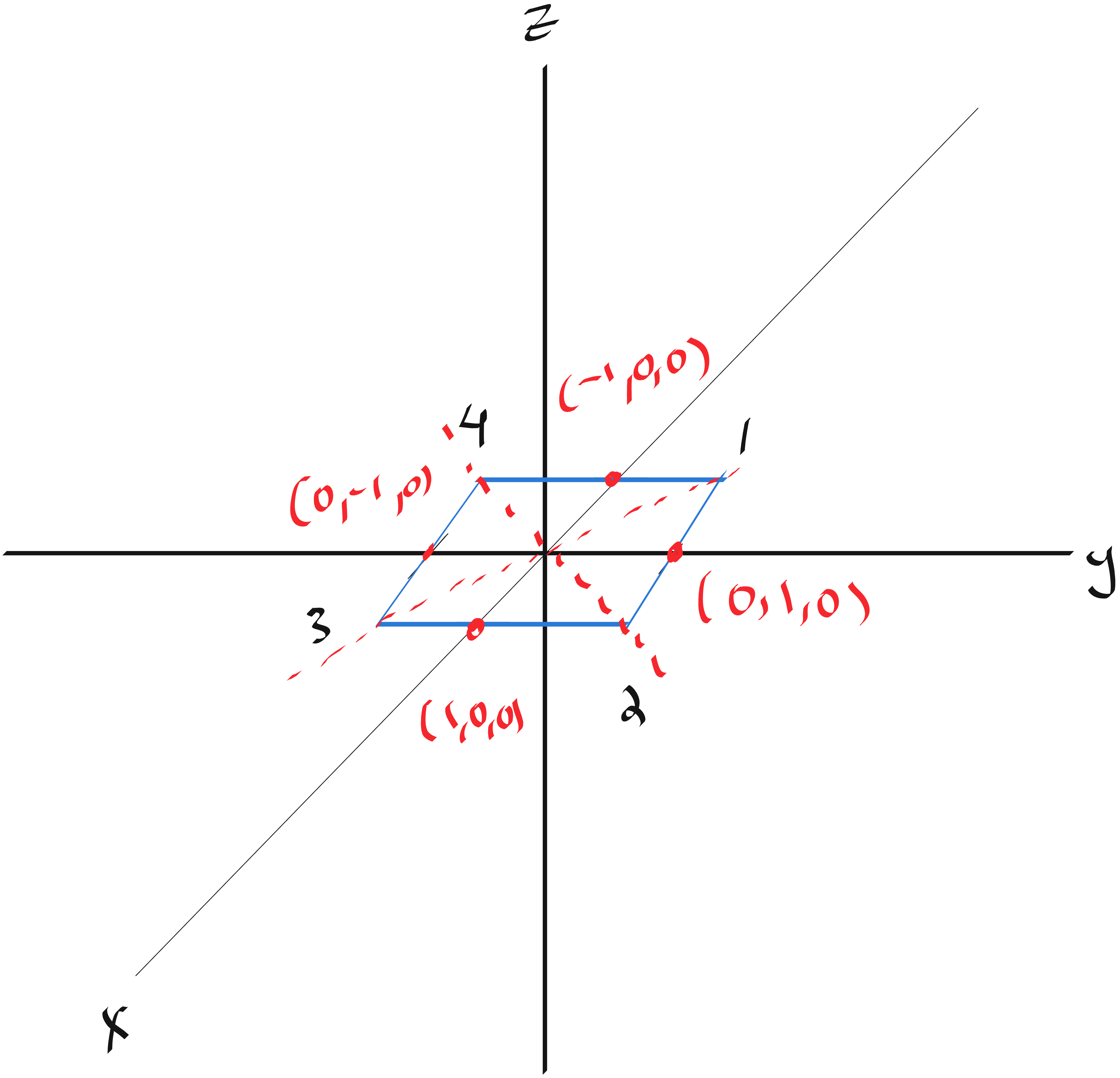
You win with switching in 2/3 of the scenarios!

Moral: Tables can help!

Let's compute the "multiplication" table for the symmetries of the square.

Convention for rotation: counterclockwise

Fix a square in $\mathbb{R}^3$ as follows:



Square is in xy plane.

e = do nothing

r = rotation counterclockwise in xy plane.

A = reflection about the x-axis

B = reflection about the y-axis

C = reflection about the line containing $(-1,1,0)$ and $(1,-1,0)$

D = reflection about the line containing $(1,1,0)$ and $(-1,-1,0)$.

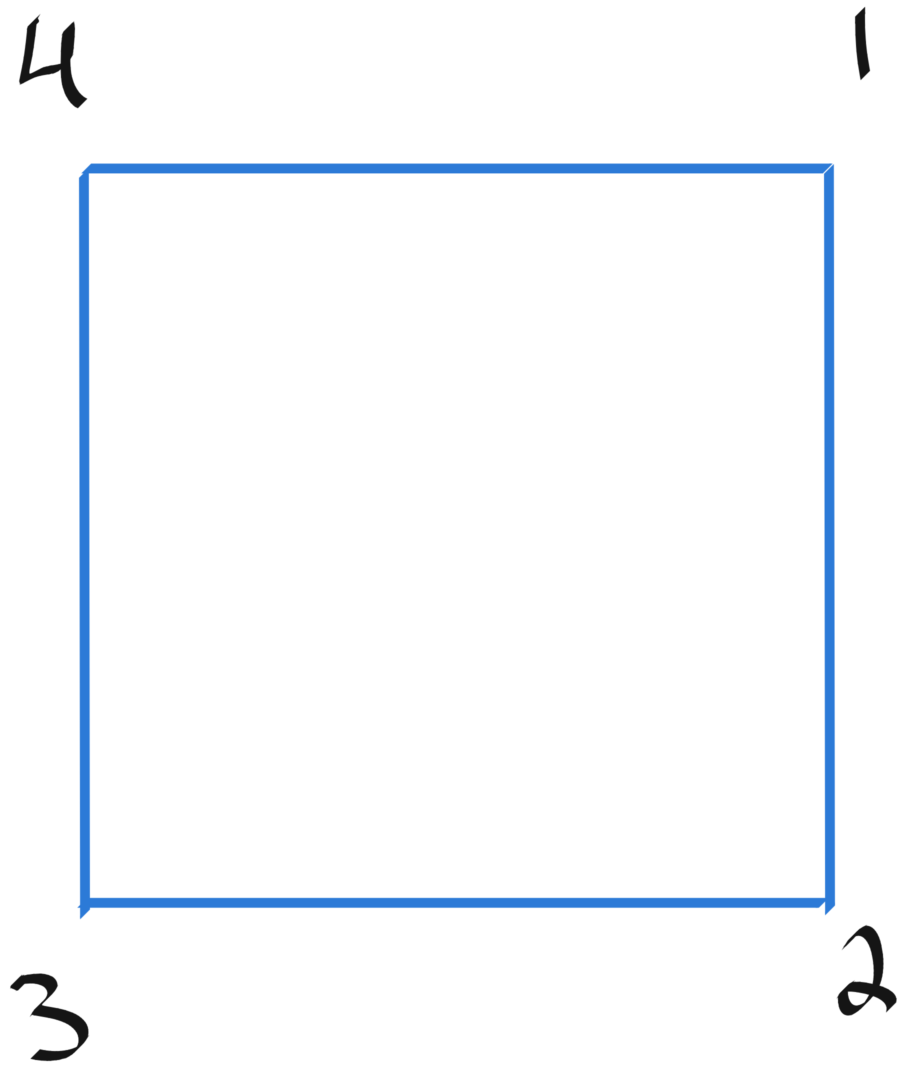
Multiplication Table

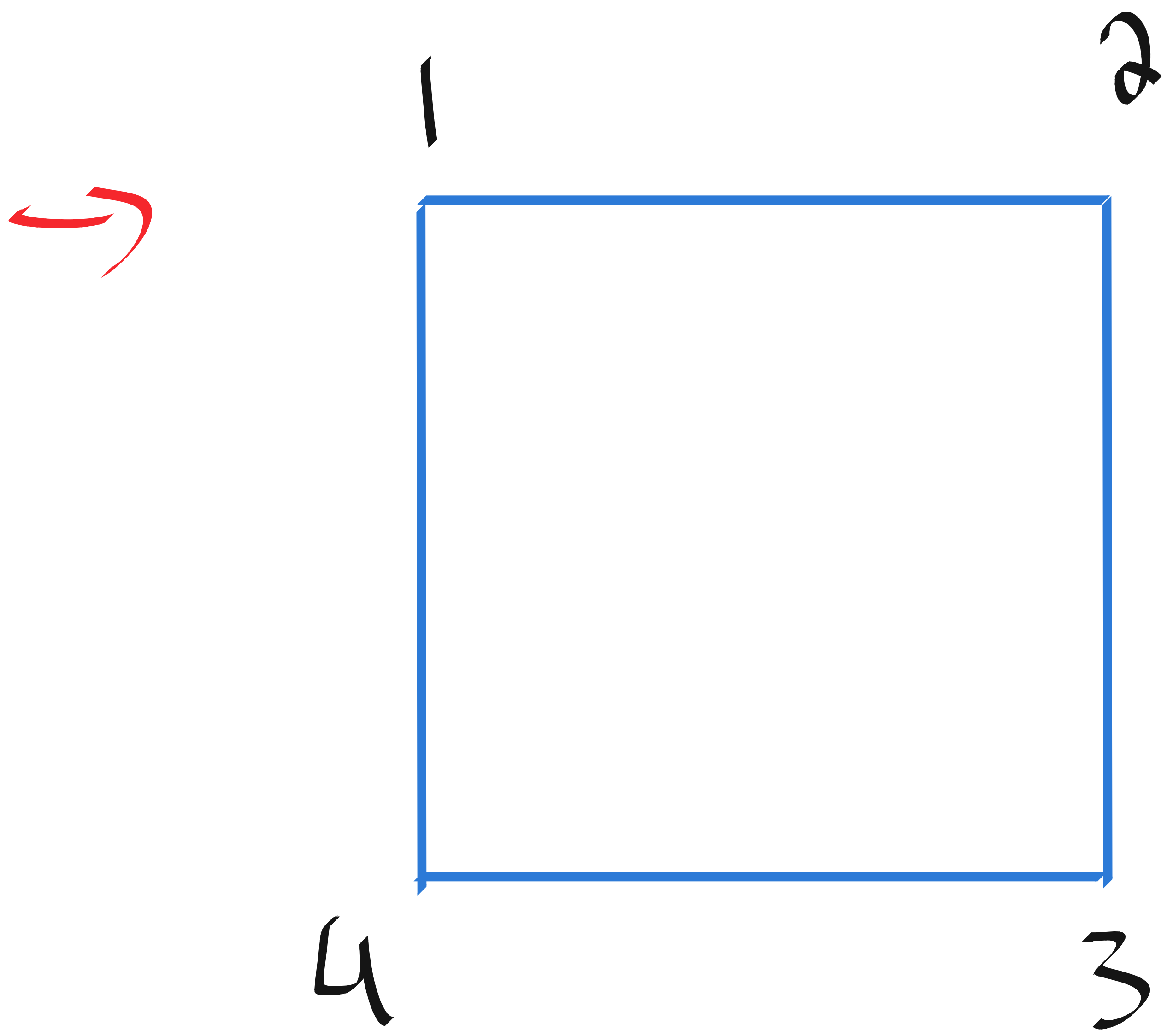
	e	r	r^2	r^3	A	B	C	D
e	e	r	r^2	r^3	A	B	C	D
r	r	r^2	r^3	e	D	C	A	B
r^2	r^2	r^3	e	r	B	A	D	C
r^3	r^3	e	r	r^2	C	D	B	A
A	A	C	B	D	e			
B	B	D	A	C	r^2	e		
C	C	B	D	A	r		e	
D	D	A	C	B	r^3			e

Convention: an entry is determined by "multiplying" top first, then side, lexicographically

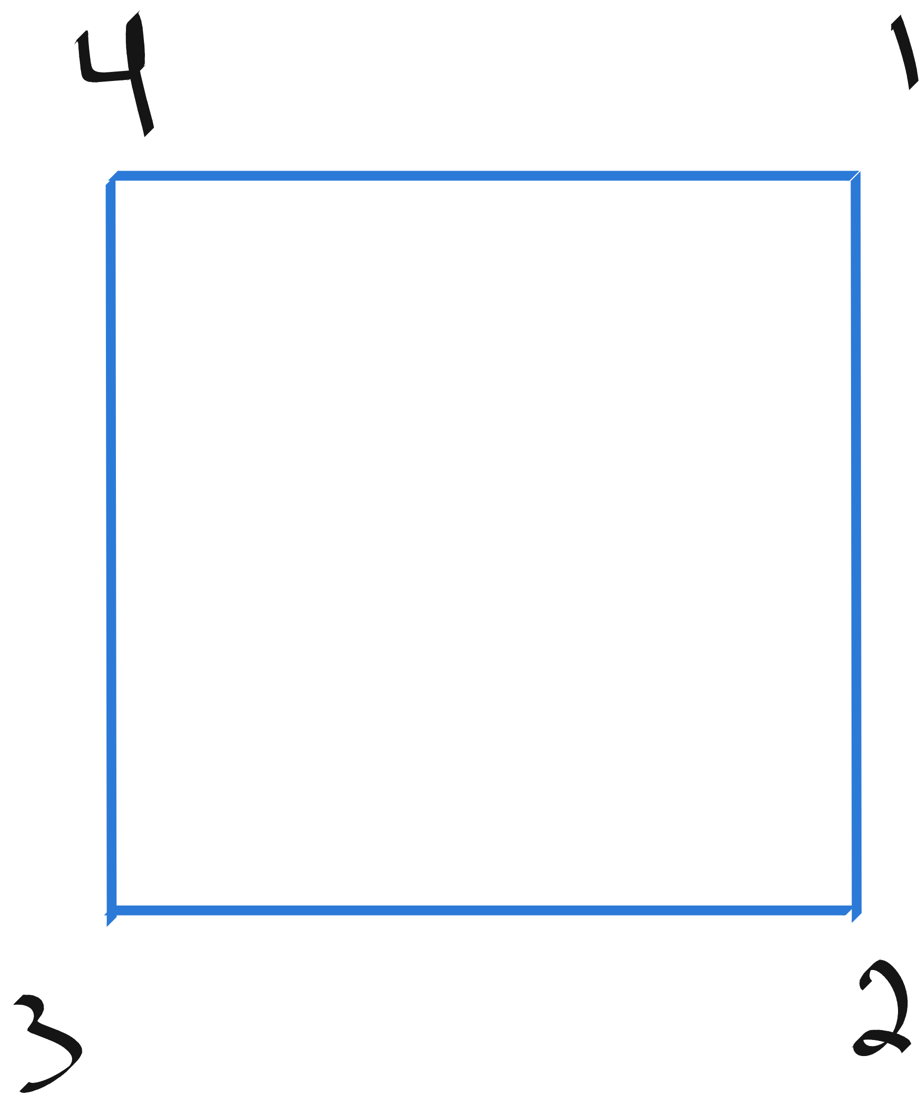
Label vertices

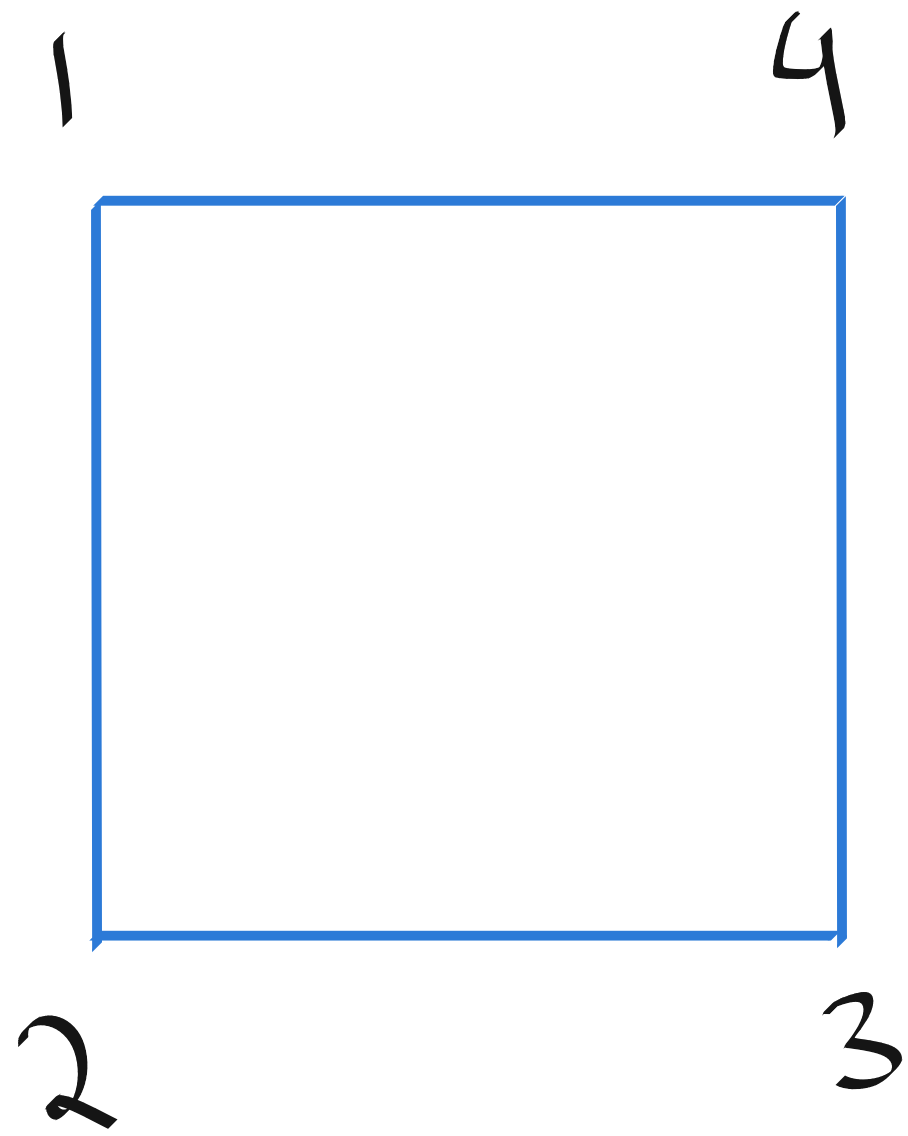
r :



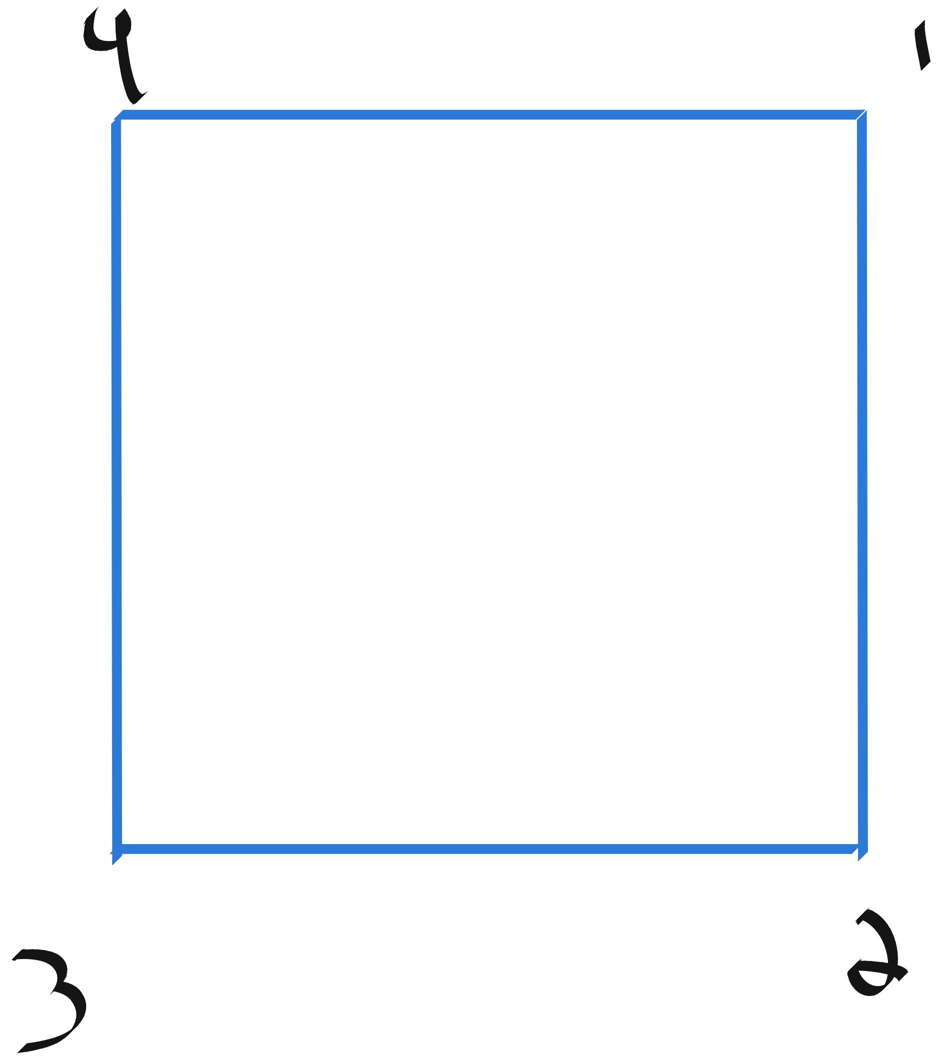


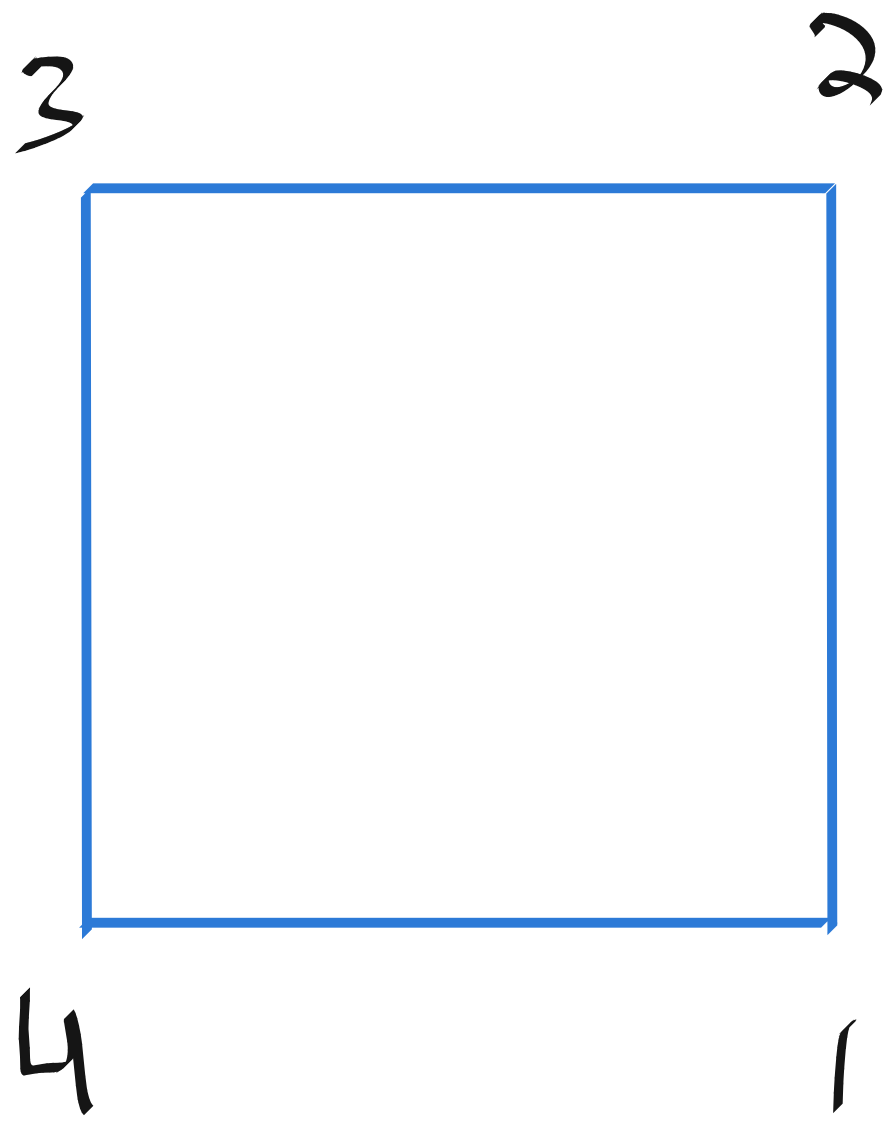
A:



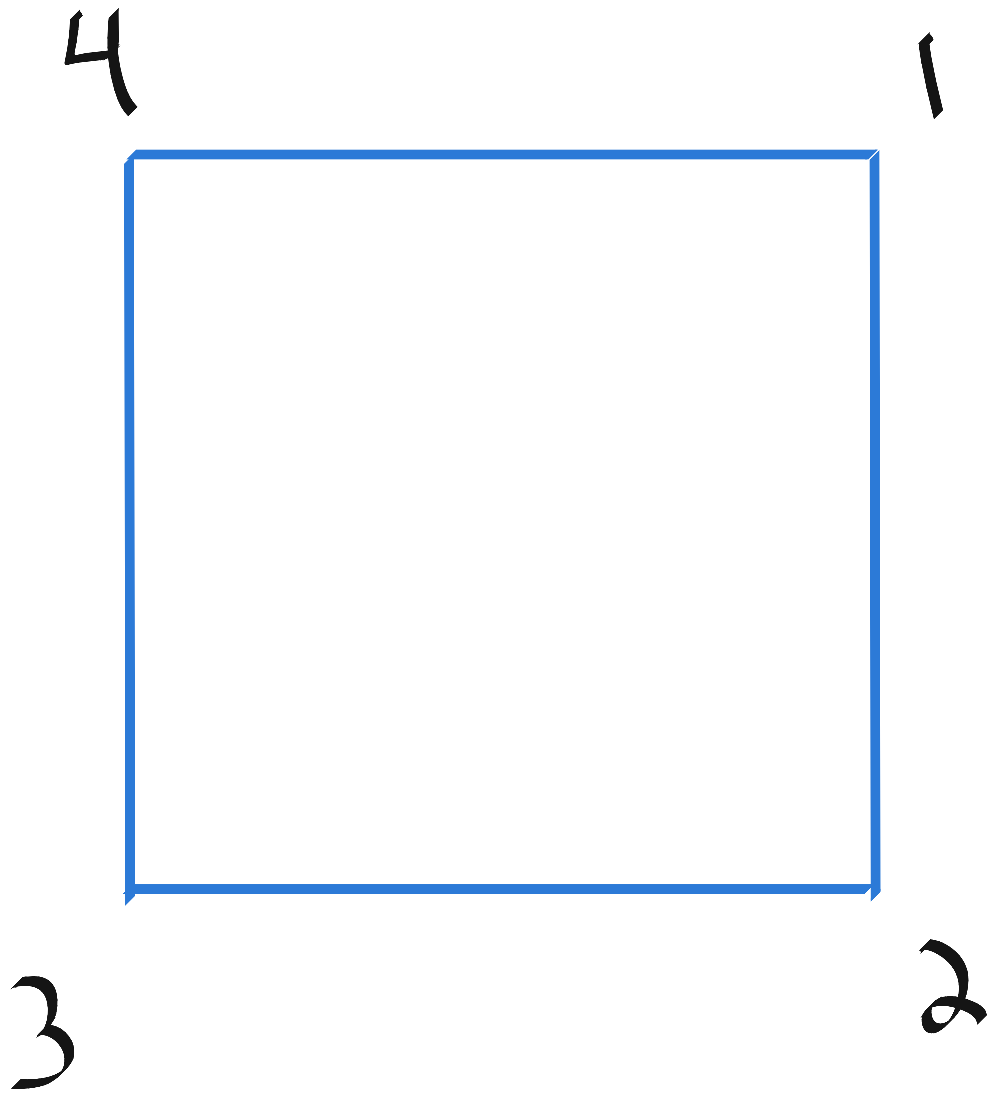


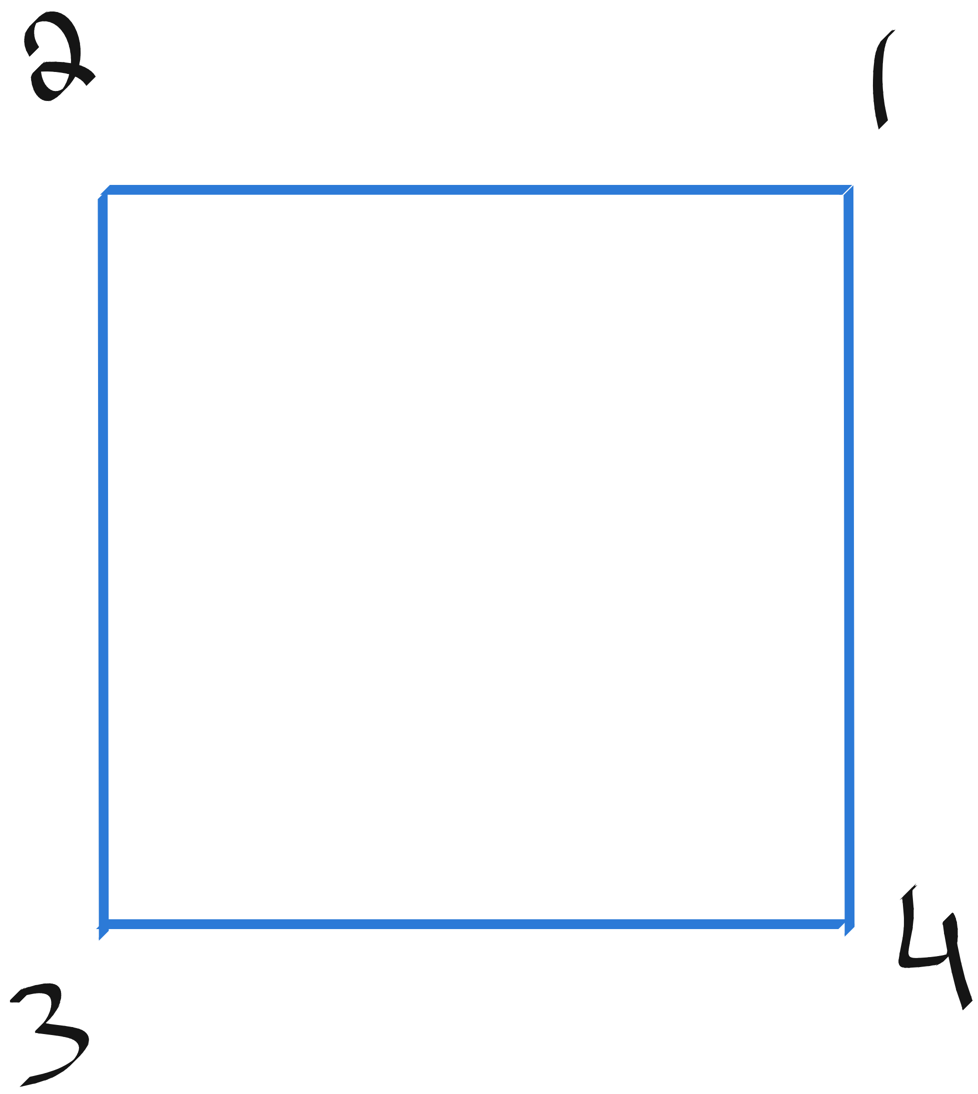
B :



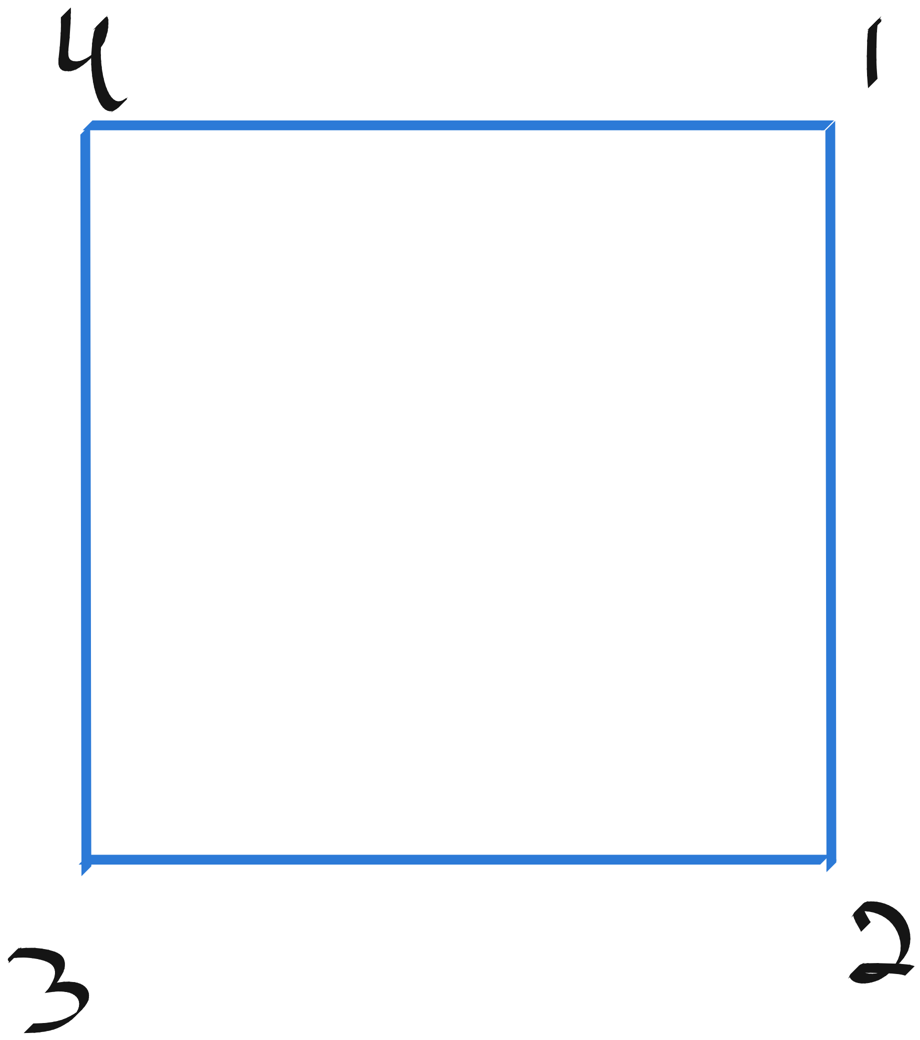


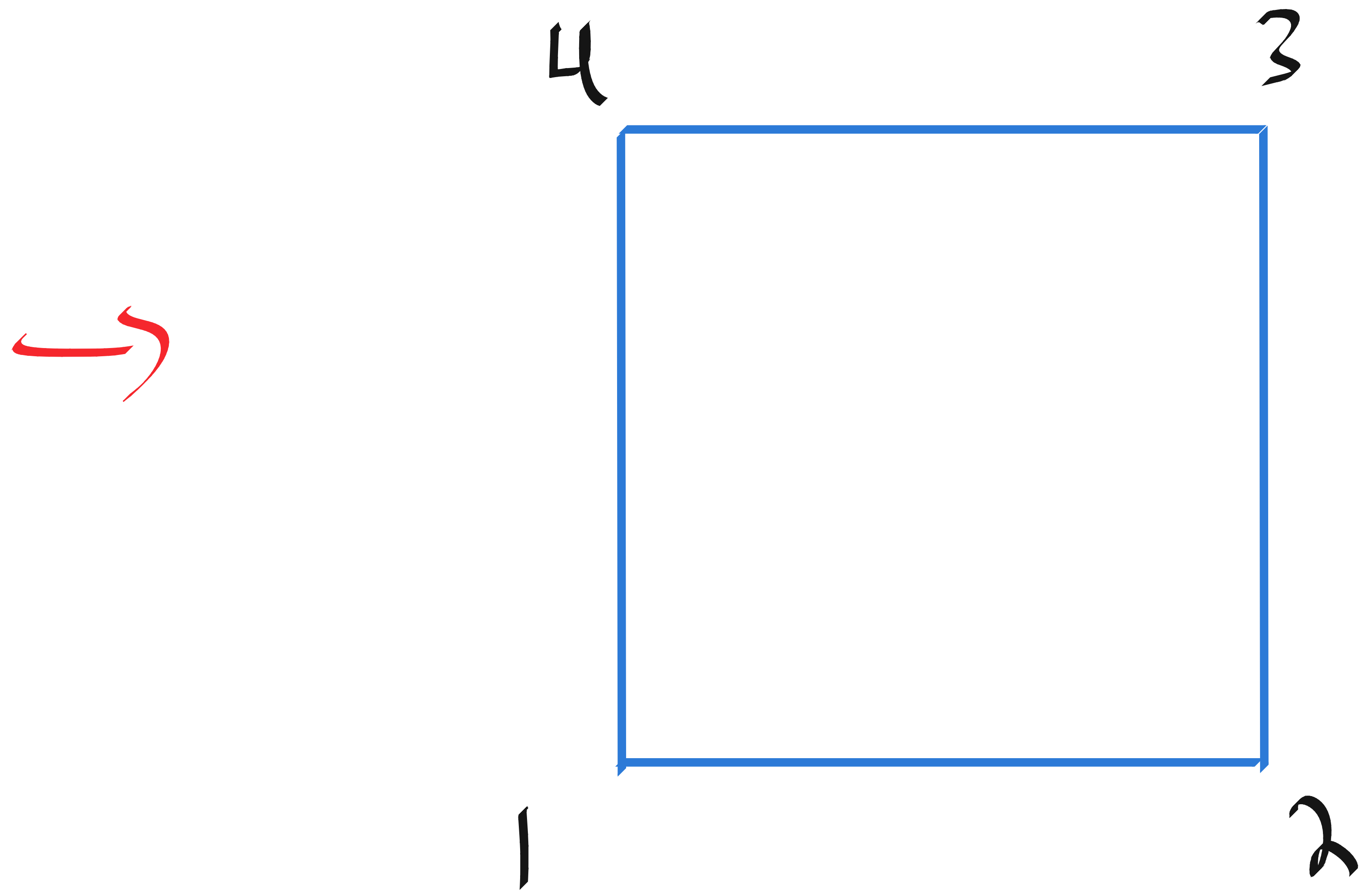
C :



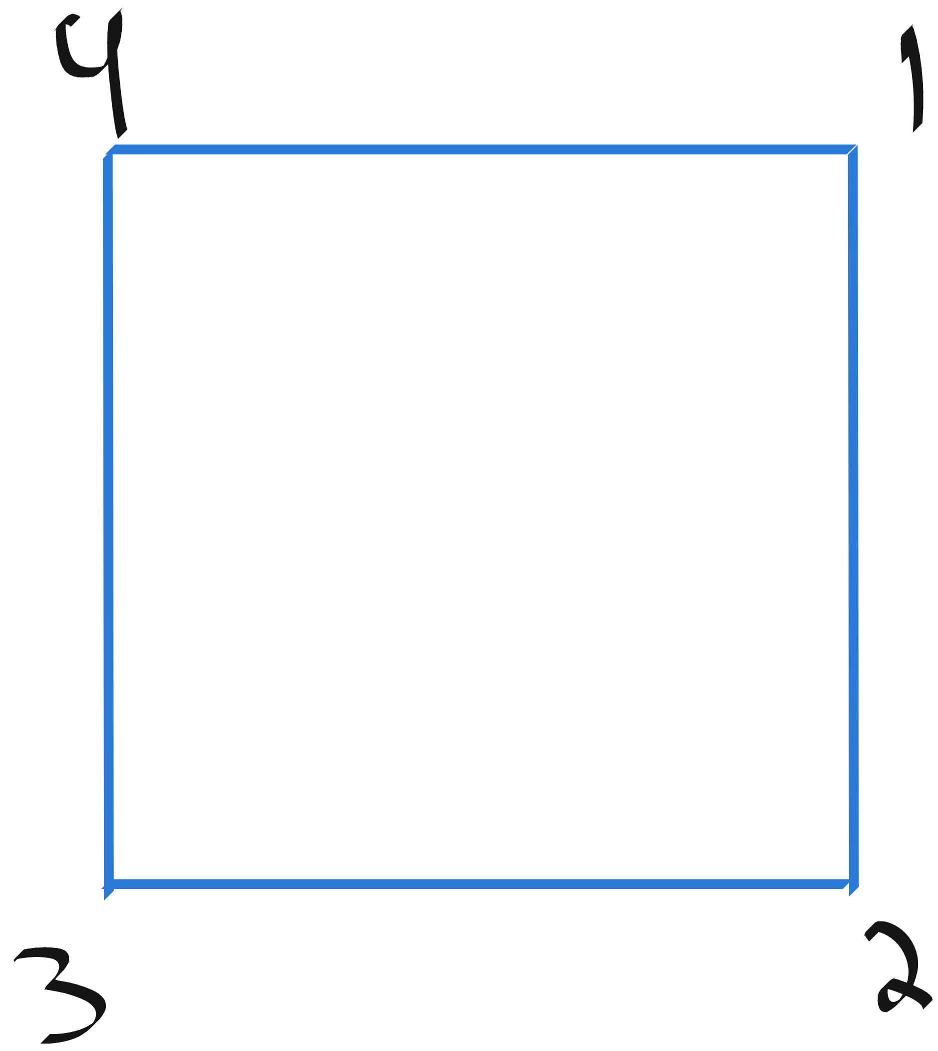


D :

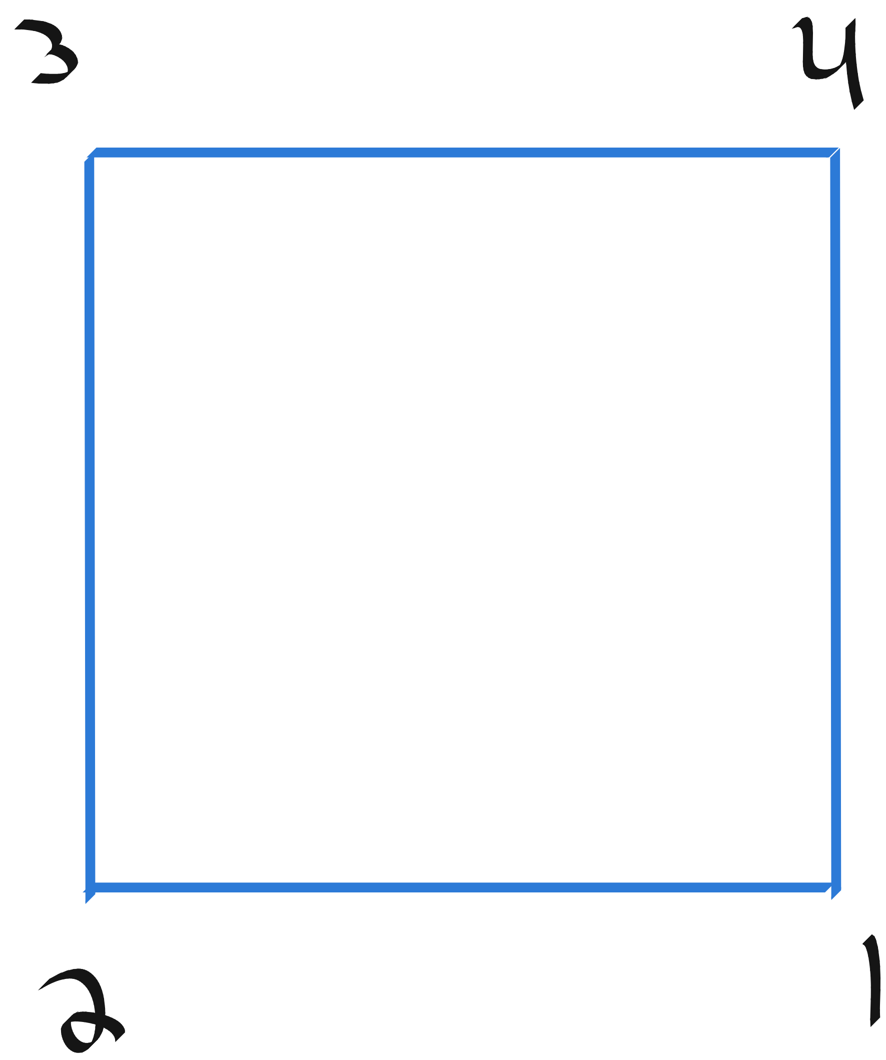




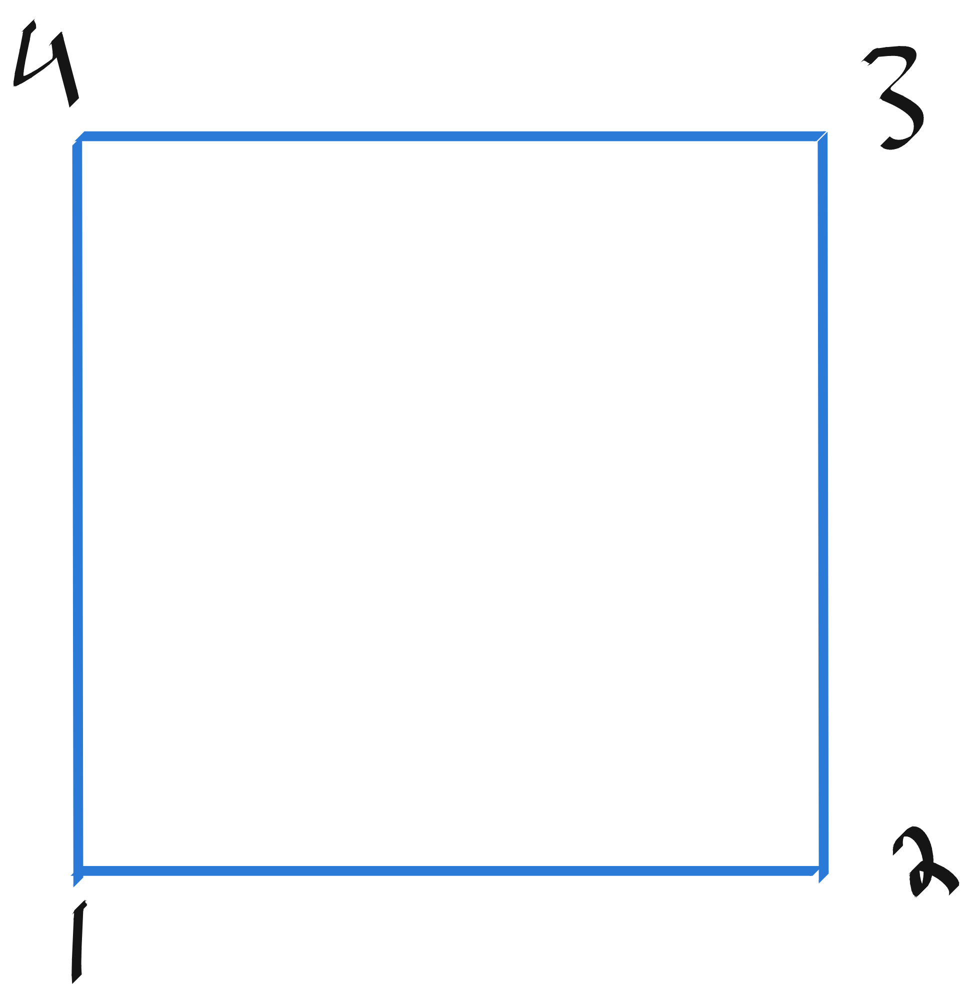
Compute Ar :



r

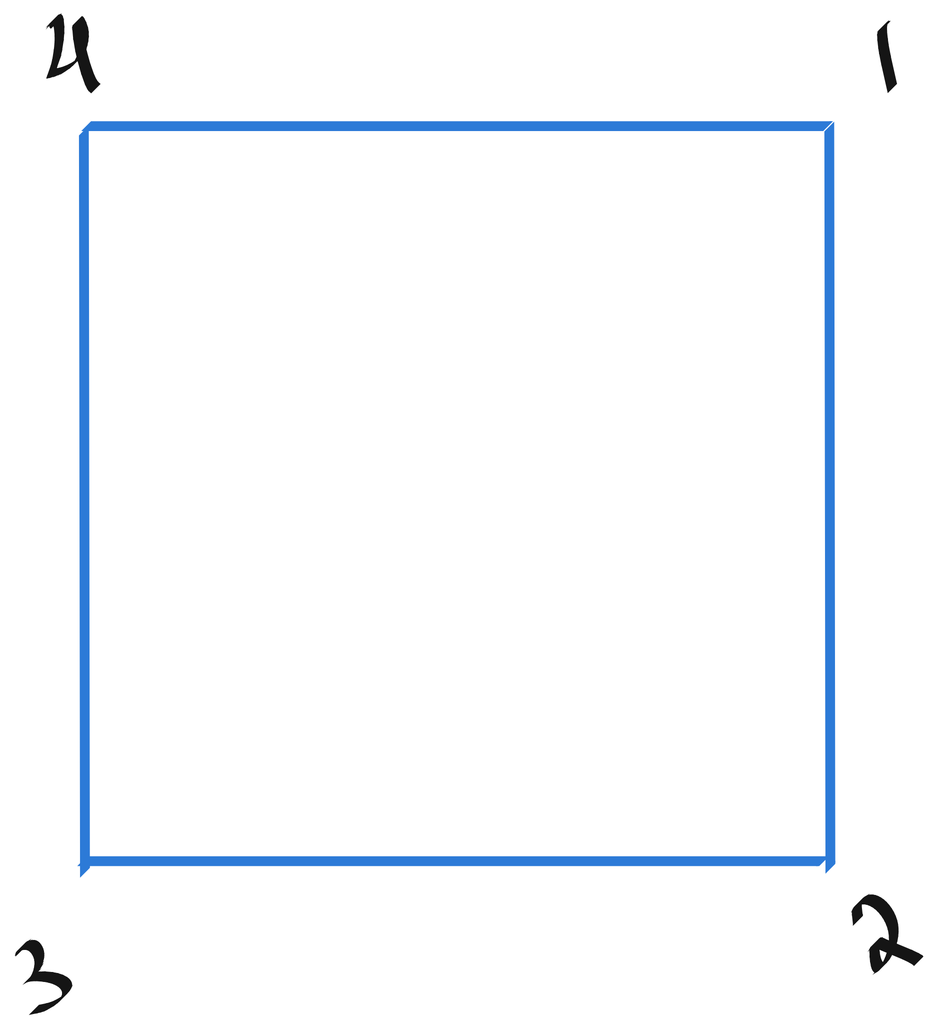


A

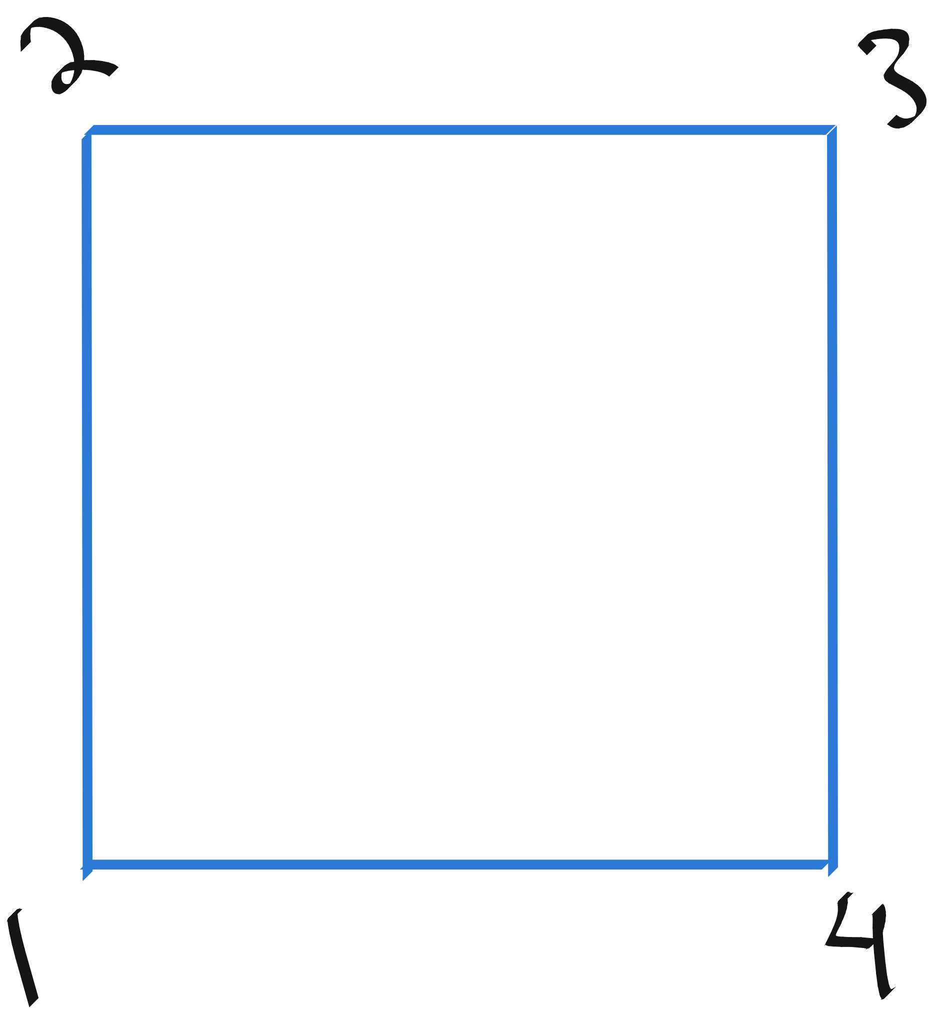


$Ar = D$

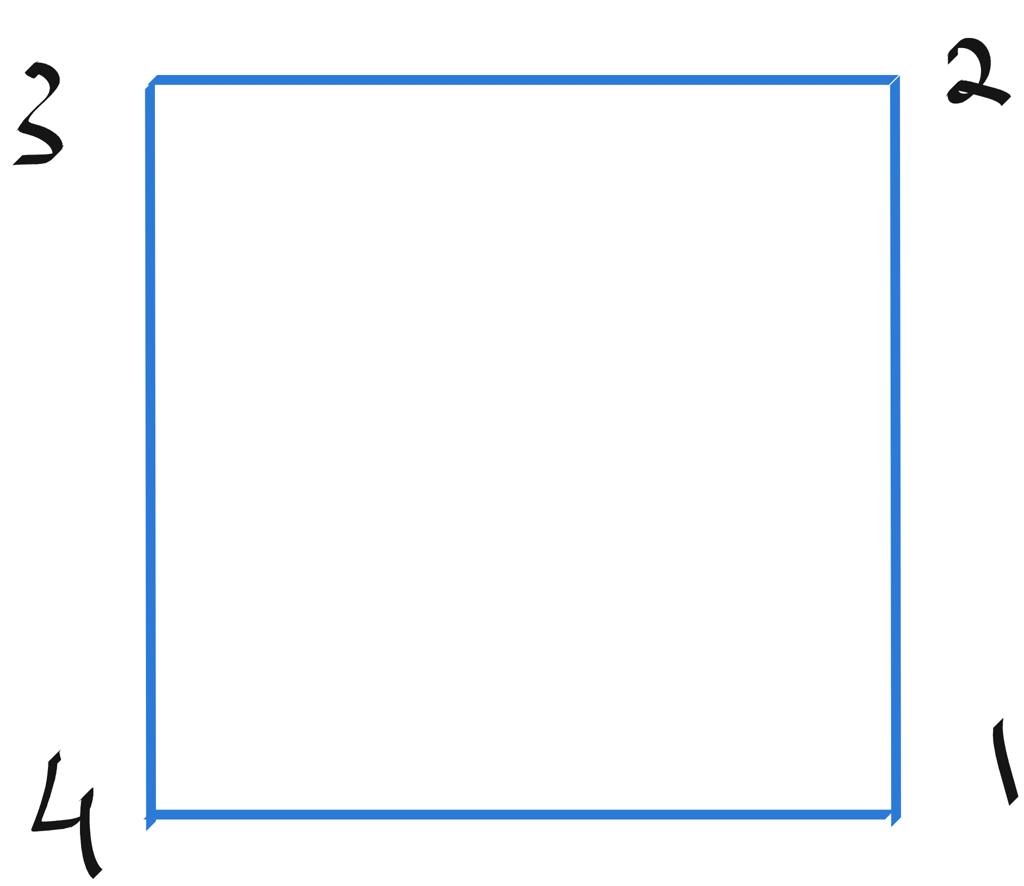
Compute $Ar^2 =$



r^2

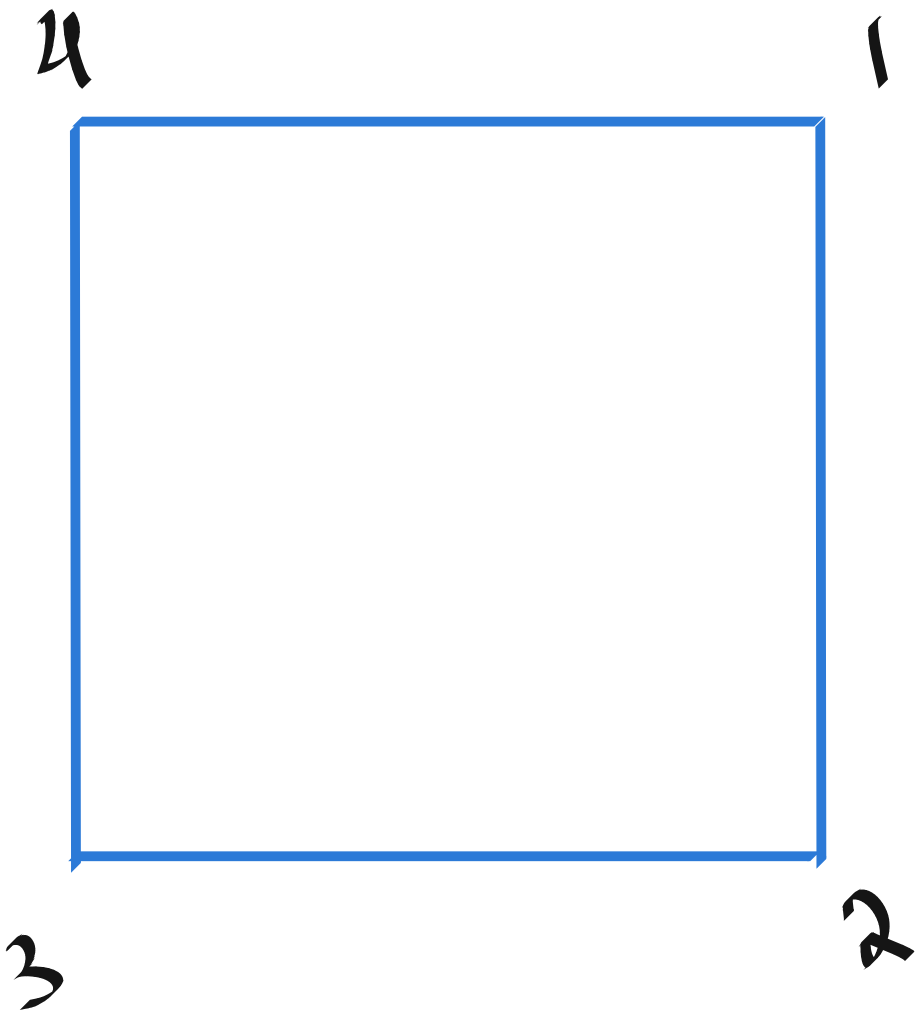


A

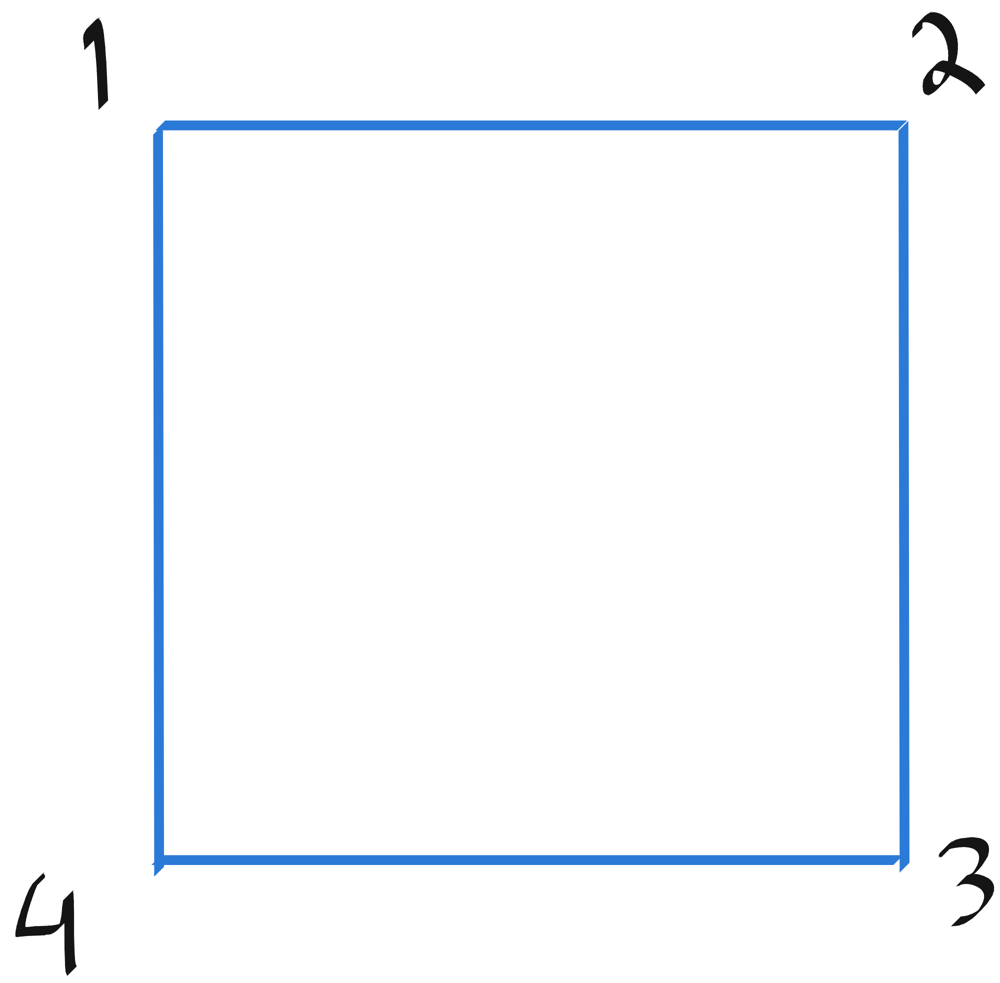


$Ar^2 = B$

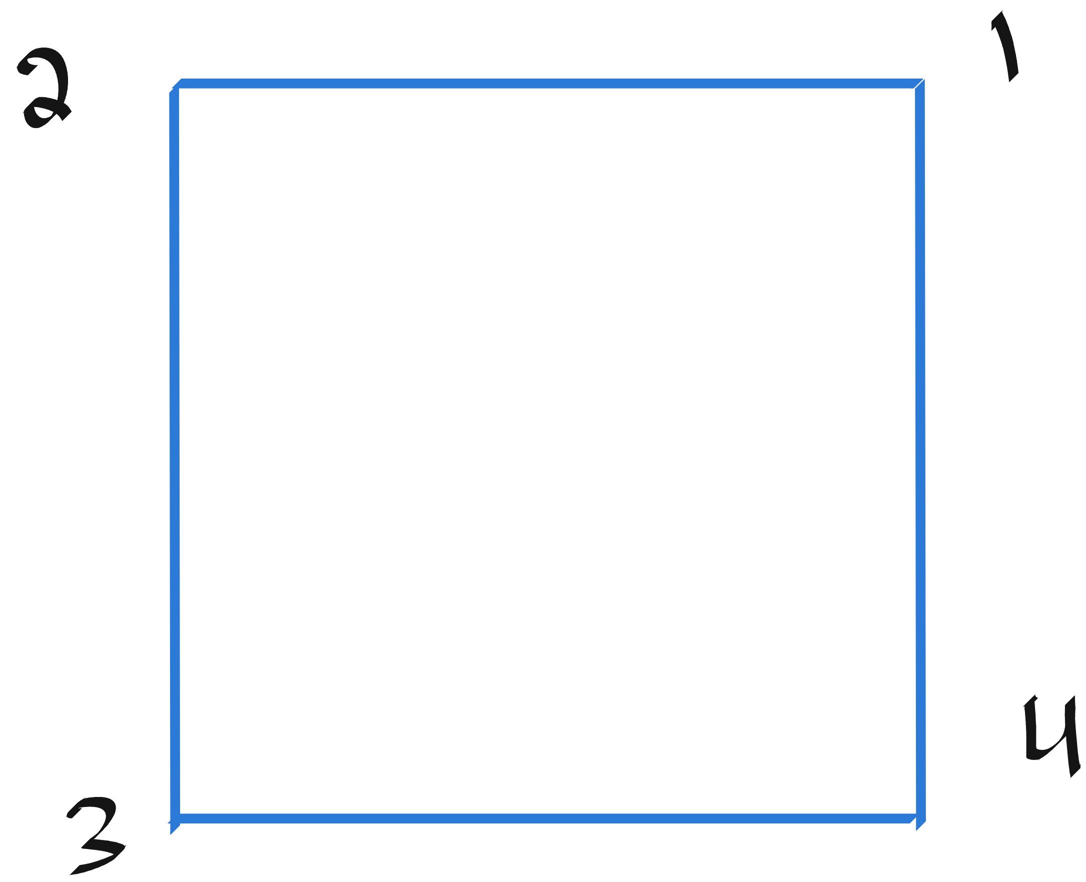
Compute Ar^3:



r^3

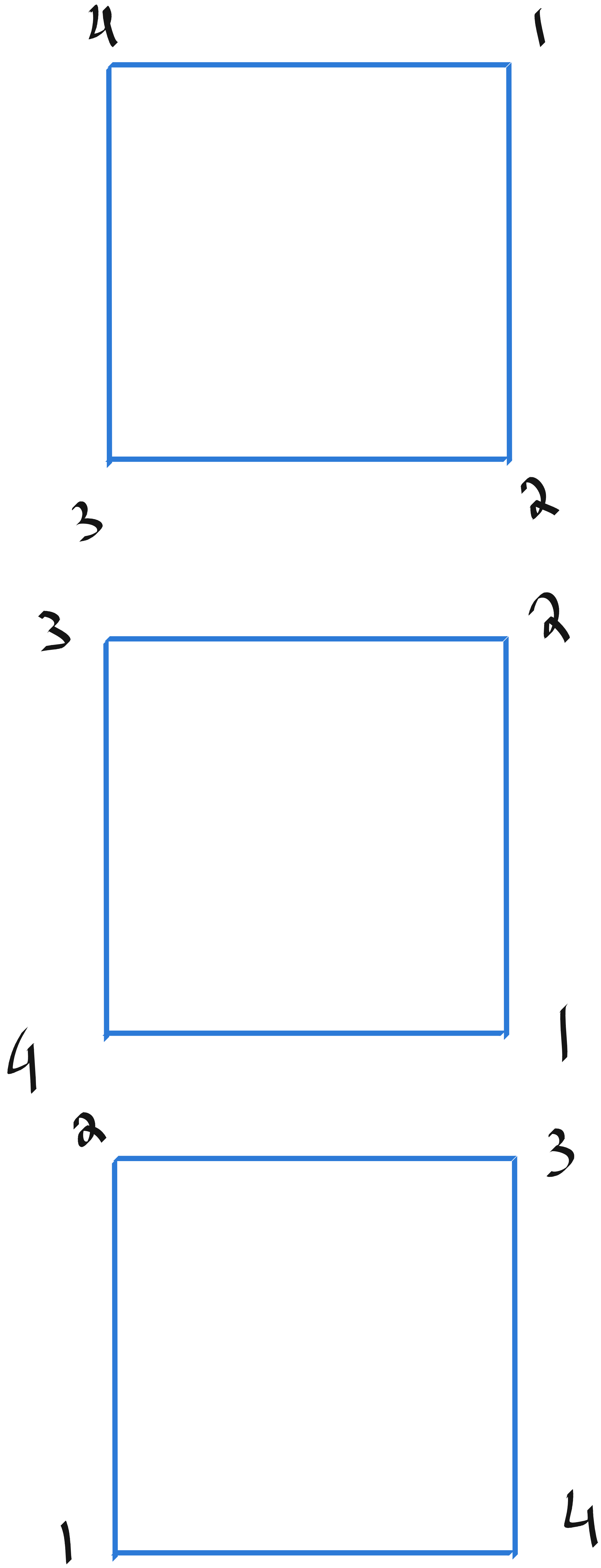


A



$$Ar^3 = C$$

Compute AB:

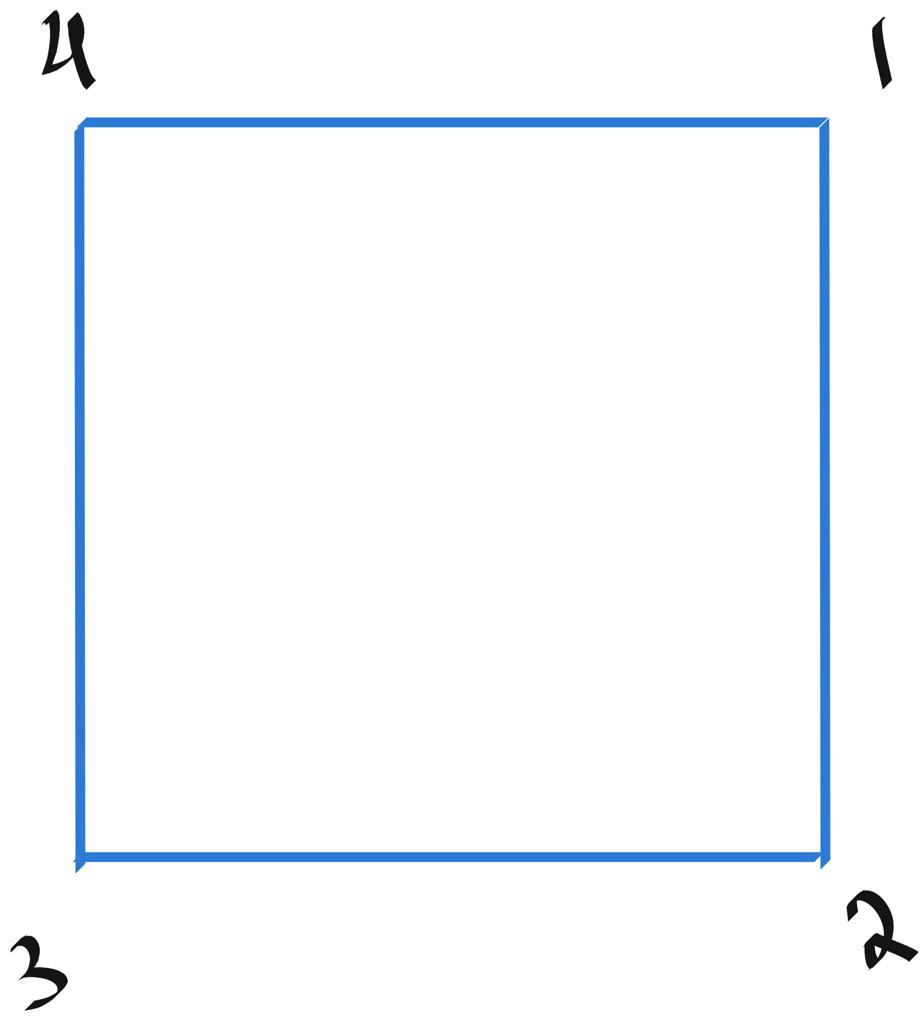


B

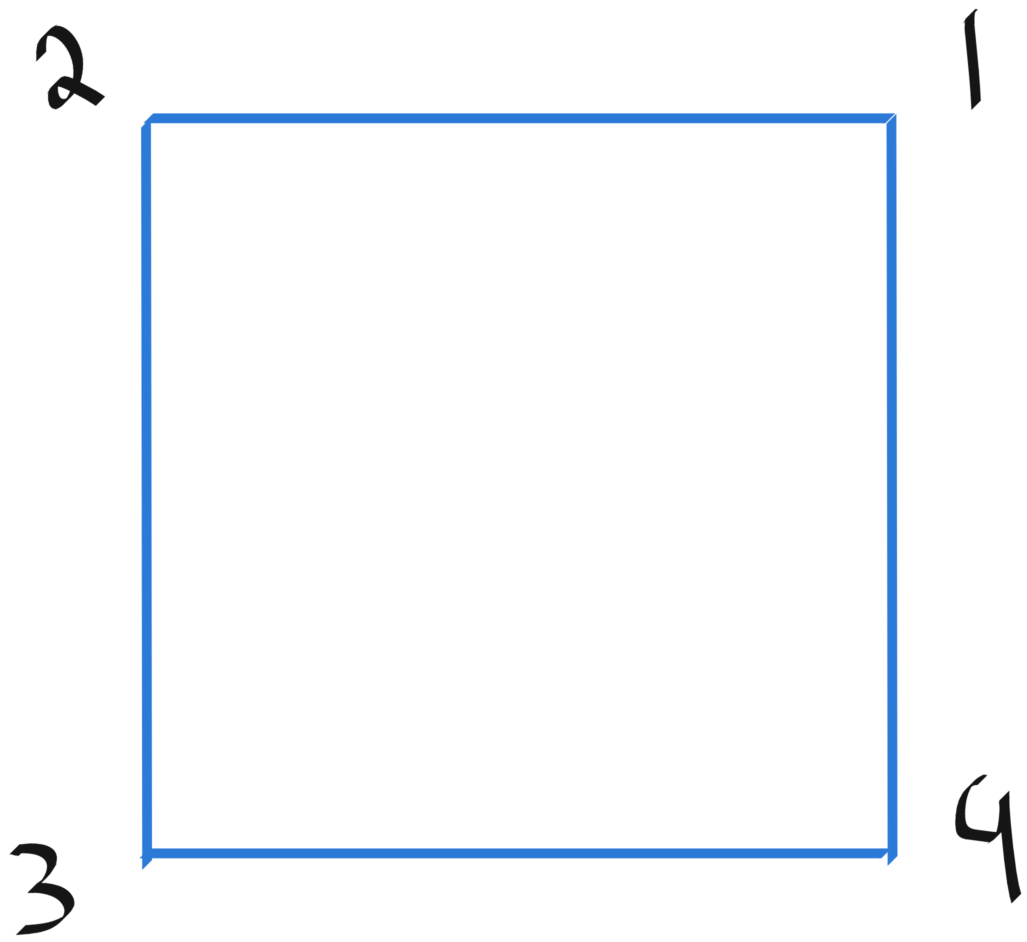
A

$AB = r^2$

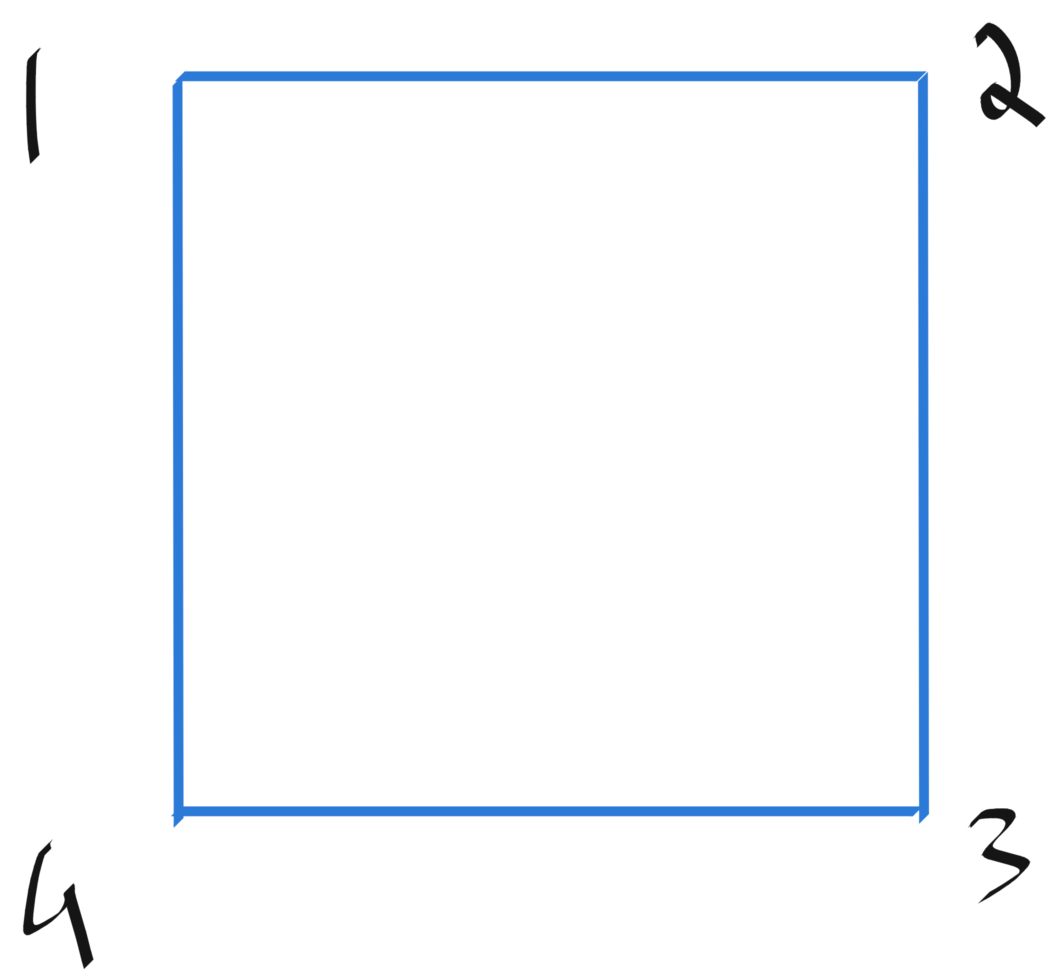
Compute AC:



C



A



$AC = r$

By process of elimination, $AD = r^3$

because if $AD = AS$ where

S is any other symmetry,

flip again by A:

$$A(AD) = A(AS)$$

function composition is associative

$$(AA)D = (AA)S$$

$$(e)D = (e)S$$

$$D = S$$

Every single symmetry has an inverse: another symmetry S which when composing, gives the identity (do nothing) symmetry. For these symmetries, it is clear that a left inverse (do the inverse second) is the same as a right inverse (do the inverse first). Will this hold true for more general symmetries?