Polynomials

(Section 1.8)

Goal: To develop division and factorization for polynomials akin to what happens in $\mathbb{Z}$. But what kind of polynomials?

Definition: (polynomials over a field)

Let K be a field. Denote by $K[x]$ the set of polynomials over K. That is, elements of $K[x]$ are of the

$$\boxed{\sum_{i=0}^{n} a_i x^i}$$

where $n \in \mathbb{N} \cup \{0\}$, $a_i \in K$ $\forall$ $0 \leq i \leq n$, and x is an indeterminate.

Addition and Multiplication in $K[x]$

Let $p(x) = \sum_{i=0}^{n} a_i x^i \in K[x]$,

$$q(x) = \sum_{i=0}^{m} b_i x^i \in K[x]$$

and suppose $n \geq m$.

Then we define

$$\boxed{p(x)+q(x) = \sum_{i=0}^{m} (a_i+b_i)x^i + \sum_{i=m+1}^{n} a_i x^i}$$

$$\boxed{p(x) \cdot q(x) = \sum_{i=0}^{m} \sum_{j=0}^{n} (b_i \cdot a_j) x^{i+j}}$$

from definition, we can see that these are binary operations on $K[x]$.

Proposition: (ring properties)

Let K be a field, $p(x), q(x), r(x)$ elements of $K[x]$.

1) Addition is commutative and associative.

2) Every element of $K[x]$ has an additive inverse.

3) Multiplication is commutative and associative.

4) $p(x) = 1$, the multiplicative identity of K, is a multiplicative identity for $K[x]$.

5) Multiplication distributes over addition