

Symmetries

(Section 1.1)

Start with any object you like. How many ways can you manipulate the object so that no observer could notice what you did?

This is a **symmetry**: an undetectable motion of an object.

Q: How should we consider the object?

In general, we'll consider objects
living in at least 3 dimensional
space.

Symmetries of Rectangles and Squares

(Section 1.2)

Start: a plain rectangular sheet of paper



Q: How many different symmetries are there for the piece of paper?

Let's count them:

1) flip on short side

2) flip on long side

3) do nothing (don't move the paper)

4) 180° rotation

Convention: the rotations will not count as distinct up to and after 360°

Similarly, two flips on the same side will count as "do nothing".

It seems there are 4 symmetries!

What if the paper is square?



List of Square Symmetries

The square has all the symmetries of a rectangle, plus

5) rotate by 90°

6) rotate by 270°

7) flip about a diagonal

8) flip about the other
diagonal

8 symmetries, $8 = 2 \cdot 4$

$= 2 \cdot (\text{number of sides})$

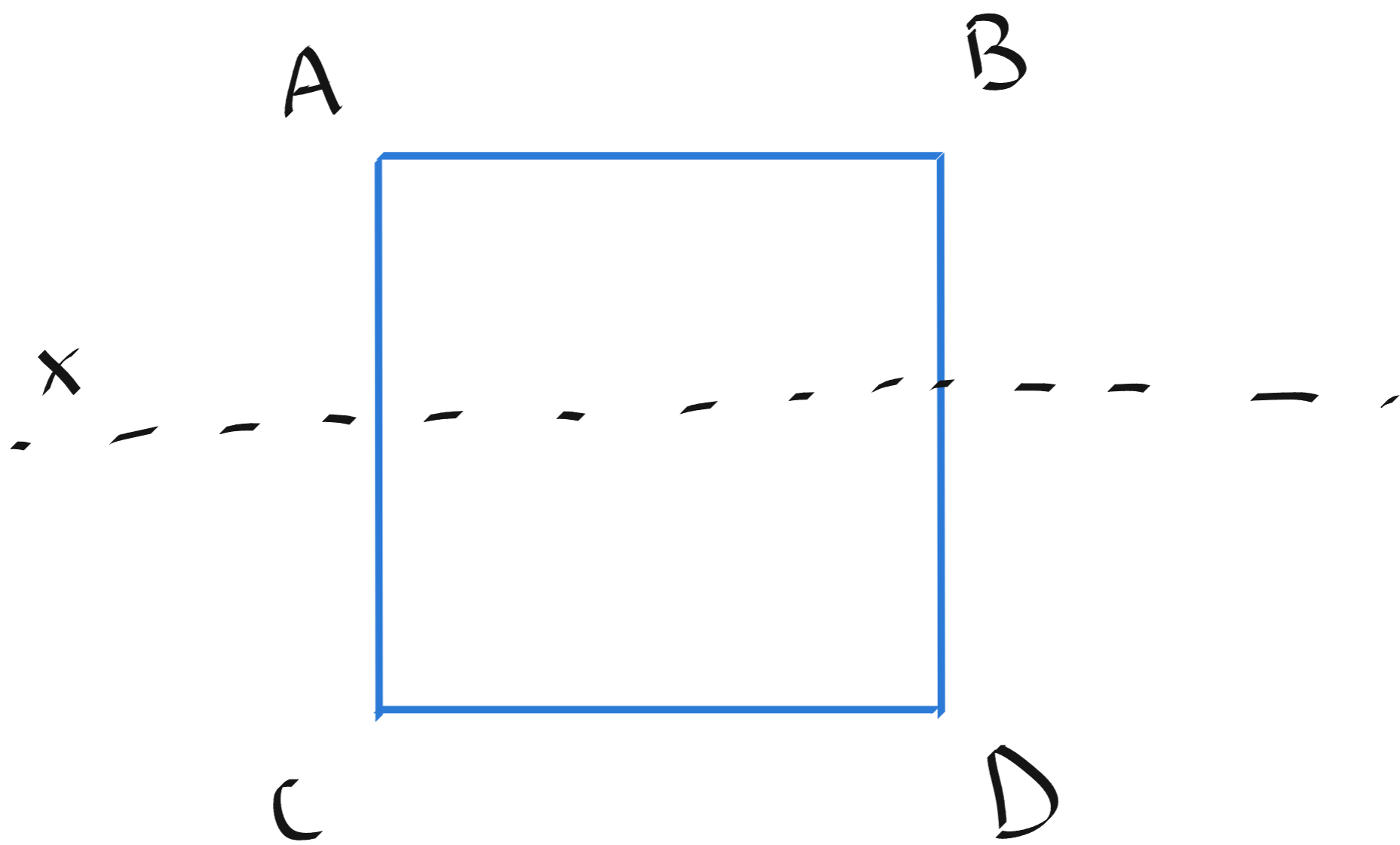
Q: What happens when you do one symmetry, followed by another?

A: You get another symmetry!

It could be that repeated symmetries eventually produce the same symmetries as used.

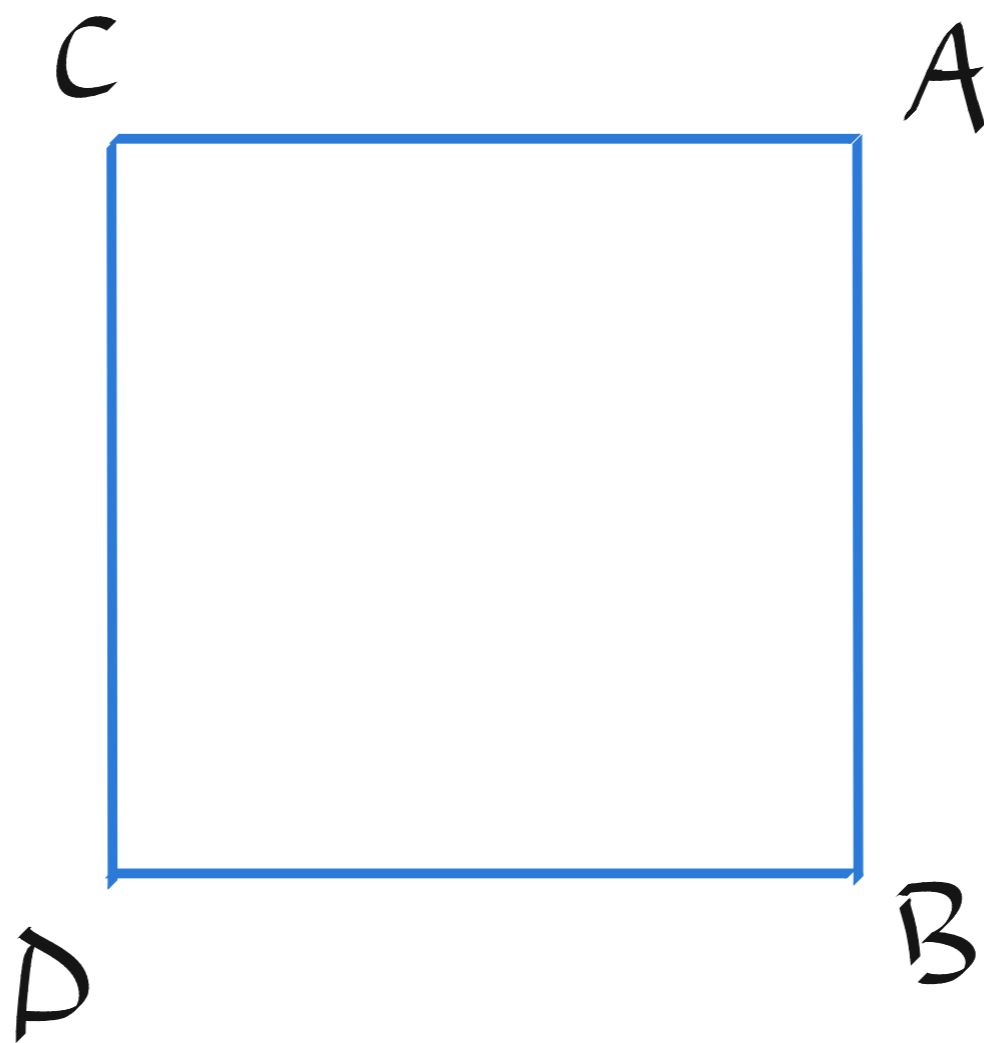
Q: Does the order make a difference?

To figure out the answer, let's label.

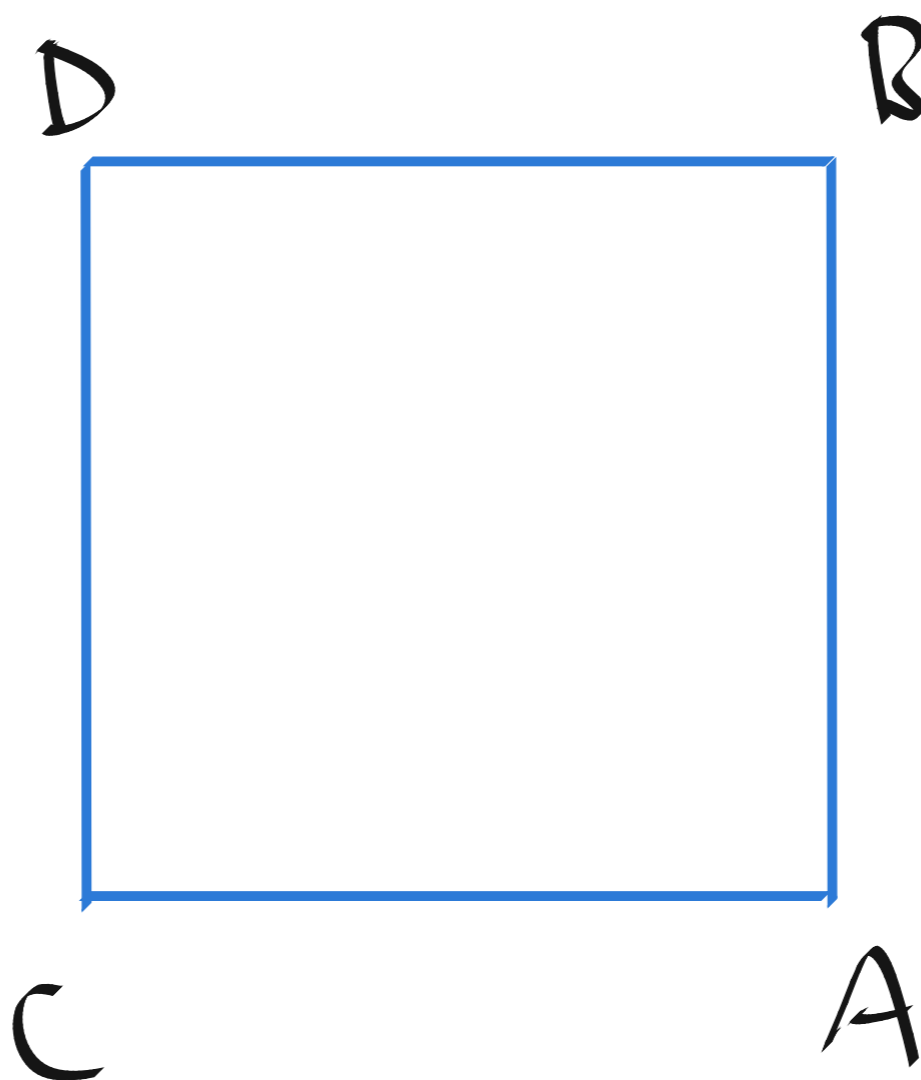


Let's rotate by 90° , then flip about X and see where the vertices go.

Rotate :

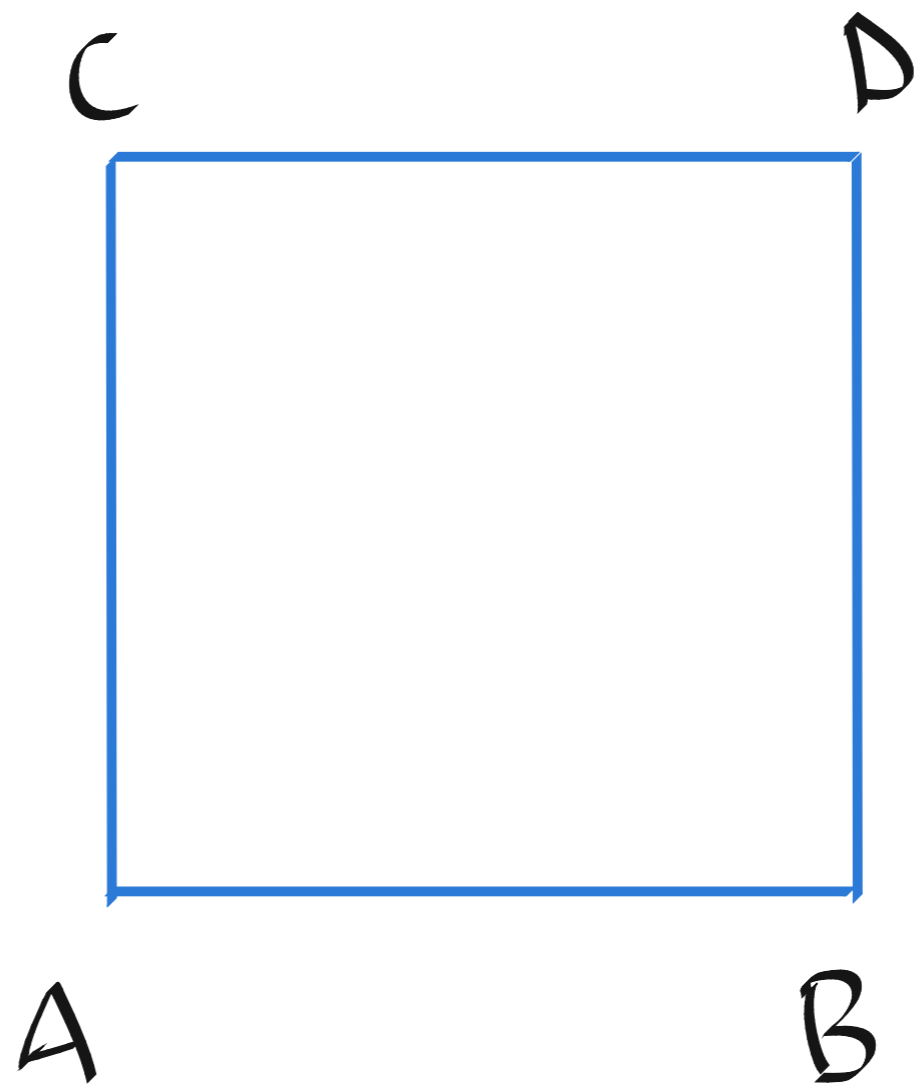


Flip :

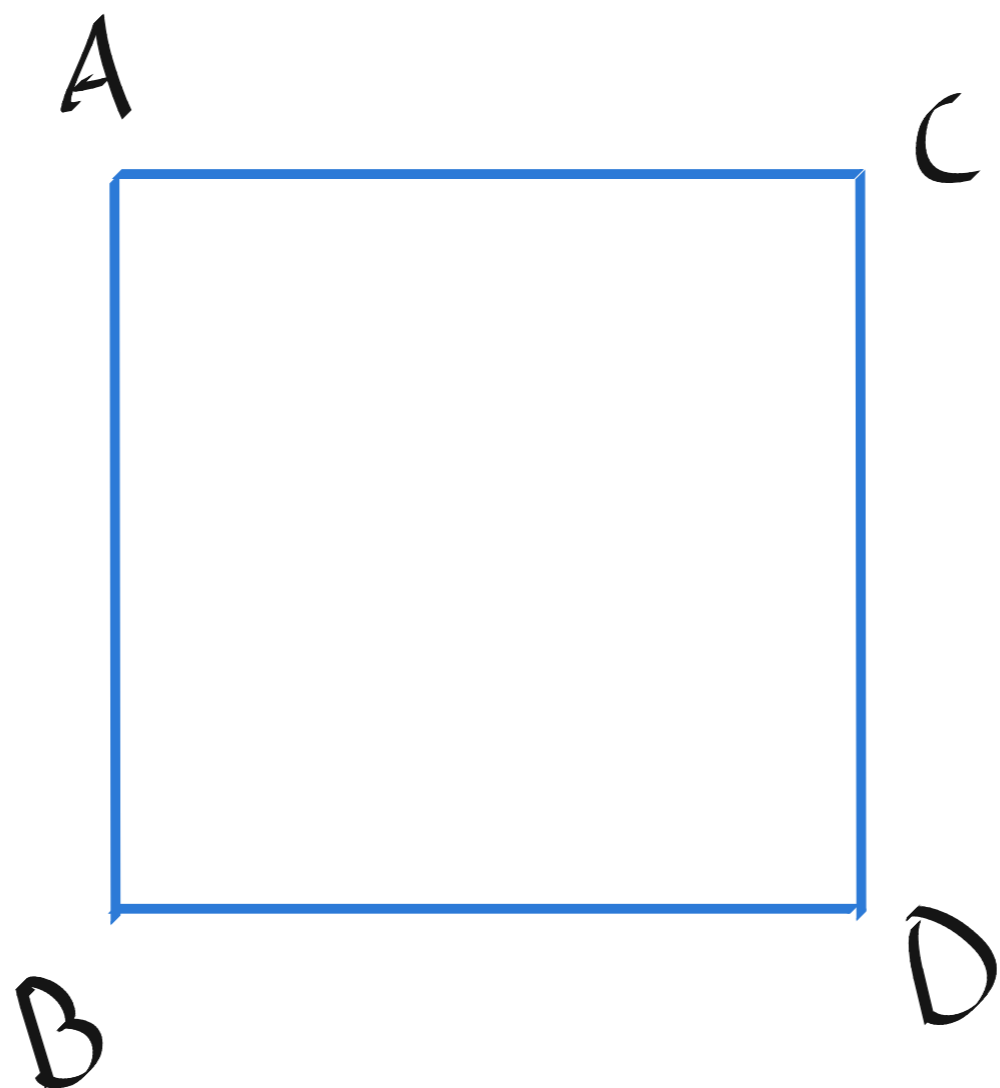


Let's do this in the opposite order:

Flip:



Rotate:



The vertices end up in different positions!