First Isomorphism Theorem:

Let G, H be groups and $\phi: G \to H$ be a surjective homomorphism. Then for $K = \ker \phi$, $G/K \cong H$.

Consider G/K the collection of left cosets of K in G and the function $\Psi: G/K \longrightarrow H$ where $\Psi(gK) = \phi(g)$

Let $g, h \in G$. Then $gK, hK \in G/K$

Suppose $\Psi(gK) = r$ and $\Psi(gK) = s$
$\Rightarrow \phi(g) = s$ and $\phi(g) = r$
$\Rightarrow r = s$, so Ψ is well-defined.

Let $p \in H$, then there exists $g' \in G$ such that $\phi(g') = p$ since ϕ is surjective. Also $g' \in g'K$
so $\Psi(g'K) = \phi(g') = p$ so Ψ is surjective.

Let $\Psi(gK) = \Psi(hK)$, then $\phi(g) = \phi(h)$, so $e = \phi(g)\phi(h)^{-1}$
$= \phi(g)\phi(h^{-1}) = \phi(gh^{-1})$

So $gh^{-1} \in K$ and $gK = hK$
so Ψ is injective

$\Psi(gKhK) = \Psi(ghKK) = \Psi(ghK)$
$= \phi(gh) = \phi(g)\phi(h)$
$= \Psi(gK)\Psi(hK)$
So Ψ is a homomorphism.

Since Ψ is a bijective homomorphism from G/K to H

$G/K \cong H$.