Math 452/552 Problem Set 5

To Be Presented Wednesday, April 17

1) Any 10-point problem no one did from Rudin, Chapter 9 on the last homework: still 10 points each! These are #'s 9, 12, 14, 15, 17, 18, 21-23, 26-29

2) Spivak problems 4-2, 4-3, 4-5, 4-11, 4-13, 4-14, 4-15: 5 points each

3) Spivak problems 3-37(b), 4-1, 4-5, 4-6, 4-7, 4-8, 4-10, 4-12, 4-17, 4-18, 4-25, 4-27, 4-30, 4-31: 10 points each

4) Spivak problems 3-38, 4-19 and 4-20, 4-28, 4-32: 15 points each (4-19 is all-or-nothing)

5) (15 points) Let $\Gamma := \{(x,y,z) \in \mathbb{R}^3 : e^{xy} = x, x^2+y^2+z^2 = 10\}$. The Implicit Function Theorem ensures that Γ is a curve in some neighborhood of the point

$$p = \left(e, \frac{1}{e}, \sqrt{10 - e^2 - \frac{1}{e^2}}\right).$$

That is, there is an open interval $I \subset \mathbb{R}$ and a C^1 (continuously differentiable) mapping $\gamma : I \to \Gamma$ such that $\gamma(0) = p$. Find a unit vector v such that $v = \pm\dfrac{\gamma'(0)}{\|\gamma'(0)\|_2}$.

6) (15 points) Let n be an integer greater than 1 and consider the following statement: if ω is a differential 2-form on $\mathbb{R}^n$ with the property that $\omega \wedge \lambda = 0$ for every differential 1-form λ, then ω must be the zero form. For what n is the above statement true? For what n is it false? Prove your answers.

7) (15 points) Let Ω be an open set in $\mathbb{R}^2$. Let u be a real-valued function on Ω. Suppose that for each point $a \in \Omega$ the partial derivatives $u_x(a)$ and $u_y(a)$ exist and are equal to zero.

a) Prove that u is locally constant, i.e., for every point in Ω there is a neighborhood on which u is a constant function.

b) Prove that if Ω is connected, then u is a constant function.