

Math 115 Exam 1

1) (15 points) Find all horizontal and vertical asymptotes for the function

$$f(x) = \frac{61x^2 - 244}{x^2 + 3x - 10}.$$

2) Let $f(x) = \frac{2x - 3}{5x - 4}$ for $x \neq 4/5$.

a) (6 points) Using the definition of the derivative, express the slope of the tangent line to the graph of f at $x = 2$ as a limit BUT DO NOT COMPUTE THE LIMIT.

b) (10 points) Obtain the equation of the tangent line at $x = 2$ to the graph of f , using any method at your disposal other than merely your calculator.

3) Consider the function

$$f(x) = \begin{cases} -6xm + 12 & x > 3 \\ -6 & x = 3 \\ x^2m - 15 & x < 3 \end{cases}$$

a) (12 points) Find all values of m (if any exist) such that f has a limit at $x = 3$.

b) (18 points) Find all values of m (if any exist) that make f continuous at $x = 3$.

4) (12 points) Show that the function $f(x) = \cos(\pi x) - 2x$ has a zero in the interval $[-1, 1]$.

5) Evaluate the following limits.

a) (5 points) $\lim_{x \rightarrow 1} \frac{3x - 6}{|x - 2|}$

b) (10 points) $\lim_{x \rightarrow 2^+} \sin(x - 2) \cos\left(\frac{7x}{x - 2}\right)$

c) (12 points) $\lim_{x \rightarrow -\infty} (\sqrt{x^2 - x + 1} - \sqrt{x^2 + 1})$