Name:

Math 115 Exam 1

February 6, 2014

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) True/False. If the sentence is false, correct the error.

a) (2 points) If $\lim_{x \to a^+} f(x) = f(a)$, then f is continuous at $x = a$.

b) (2 points) If the limit exists, the equation of the tangent line to the graph of f at $x = a$ is given by $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$.

c) (2 points) If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, then $\lim_{x \to a}(f(x) + g(x)) = L + M$.

d) (2 points) For all f and g, if $f'(x)$ and $g'(x)$ exist, then $(f \cdot g)'(x) = f'(x) \cdot g'(x)$.

e) (2 points) The function $f(x) = \dfrac{x^2 - 3x}{x - 3}$ has a vertical asymptote at $x = 3$.

2) Find the equation for the tangent line to the given function at the indicated point, using any method at your disposal other than just your calculator.

a) (12 points) $f(x) = (x^2 - 2x + 1)(x^2 - 5x + 6)$, $a = 0$.

b) (14 points) $g(x) = \dfrac{2x - 1}{5 - 3x}$, $a = 4$.

3) Consider the function

$$f(x) = \begin{cases} 3xc^2 - 12, & x > 4 \\ 0, & x = 4 \\ 2xc^2 + x^2c + 8, & x < 4 \end{cases}$$

a) (12 points) Find all values of c (if any exist) such that f has a limit at $x = 4$.

b) (12 points) Find all values of c (if any exist) that make f continuous at $x = 4$.

4) (12 points) Show that the function $f(x) = \tan(x) + x - 1$ has a zero in the interval $[-\pi/4, \pi/4]$.

5) Evaluate the following limits.

a) (6 points) $\displaystyle\lim_{x\to -\pi/6} \frac{\cos(x)}{|\sin(2x)|}$

b) (10 points) $\displaystyle\lim_{x\to 1^+} (x-1)^2 \sin\left(\frac{1}{x-1}\right)$

c) (12 points) $\displaystyle\lim_{x\to\infty} (\sqrt{16x^2+7x-8} - \sqrt{16x^2-5x+22})$

BONUS: (10 points) For all differentiable f, find the value of

$$\lim_{h \to 0} \frac{f(ax + ah) - f(ax)}{h},$$

where a is a fixed number. Your answer should be symbolic with perhaps one exception on a. You will get zero points for finding an answer for your favorite choice of f.