Name:

Math 115 Exam 2

March 13, 2014

1) WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) True/False. If the sentence is false, correct the error.

a) (2 points) When f and g are differentiable, the derivative of $(f \circ g)$ at x is given by the formula $f'(g(x))$.

b) (2 points) Every differentiable function is continuous.

c) (2 points) A continuous function attains its maximum on a closed interval.

d) (2 points) If $f'(c) = 0$, then f has a local maximum or minimum at $x = c$.

e) (2 points) $\lim_{x \to 0} \frac{\sin(x)}{x} = 0$.

2) Calculate f' for the following functions.

a) (6 points) $f(x) = (10x^3 + 1)\cot(x)$.

b) (10 points) $f(x) = \csc(\sin(x^2))$.

3) The position of a charm quark in CERN's particle accelerator is unrealistically given by $s(t) = \dfrac{t}{2} + \dfrac{1}{t+1}$ where $t \geq 0$ is in seconds and s is in kilometers.

a) (12 points) Determine where the intervals where the quark is moving in the positive direction (increasing) or the negative direction (decreasing). Identify any local maxima or minima.

b) (8 points) Find the intervals where the quark's speed is increasing (concave up) or decreasing (concave down). Identify any inflection points.

4) (15 points) If $\tan(x + 2y - 1) - xy^3 = -\dfrac{\pi^3}{8}$, find the equation of the tangent line to the graph at the point $(1, \pi/2)$.

5) (12 points) Find the absolute maximum and minimum for the function $f(x) = (x^2 - 9)^{1/3}$ on the interval $[-1, \sqrt{10}]$

6) Evaluate the following limits.

a) (5 points) $\lim_{x\to 0} \dfrac{\sin(12x)}{5x}$

b) (10 points) $\lim_{x\to 0} \dfrac{8x}{x - \tan(2x)}$

c) (12 points) $\lim_{t\to\infty} 2t^2 \sin^2\left(\dfrac{3}{t}\right)$

BONUS: (10 points) Show that $\tan(x) > x$ for all x in the interval $[\pi/4, \pi/2)$. A picture of the graph from your calculator will get you zero points.