Name:

Math 115 Exam 3

April 10th, 2014

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations UNLESS INDICATED; just leave them as they are.

1) True/False. If the sentence is false, correct the error.

a) (2 points) If f and g are continuous on $[a, b]$,

$$\int_a^b f(x) \cdot g(x)\ dx = \int_a^b f(x)\ dx \cdot \int_a^b g(x)\ dx$$

.

b) (2 points) If f and g are continuous on $[a, b]$,

$$\int_a^b f(x) + g(x)\ dx = \int_a^b f(x)\ dx + \int_a^b g(x)\ dx$$

.

c) (2 points) The integral of a continuous function f on $[a, b]$ always gives the area between the graph of f and the x-axis from $x = a$ to $x = b$.

d) (2 points) If f is continuous on $[a, b]$, then the definite integral of f is given by $\displaystyle\int_a^b f(x)\ dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\sum_{i=1}^{n} f\left(a + \frac{i(b-a)}{n}\right)\right)$.

e) (2 points) Two continuous antiderivatives of a function f differ by a constant.

2) (15 points) Let $f(x) = \cos^4(x)$. Compute a right sum, using 8 equal subdivisions, that approximates the area between the graph of f and the x-axis from $x = 0$ to $x = 2\pi$. No decimals, I want an exact answer.

3) Sasquatch is pouring his favorite ecologically friendly beverage into an inverted right conical drinking cup at the rate of 3 in^3/s. The height of the cup is 12 inches and has a maximum radius of 4 inches at the top. The volume of a cone is $1/3\pi r^2 h$.

a) (5 points) Draw a picture that reflects this scenario, labeling your variables.

b) (8 points) Find an equation that relates the volume of the amount of liquid in the cup at a given time to the height of the liquid (no r's should appear in this equation).

c) (12 points) Determine how fast the height of the liquid is changing 2 seconds after Sasquatch begins to pour.

4) Alladin is designing a new flying carpet, which has to have a total area of 24 square feet. The carpet will have a border of cheaper material and an interior "body" of more expensive material. The border will be 9 inches on the front and back of the carpet and 6 inches on the sides.

a) (5 points) Draw a picture that reflects this scenario, labeling your variables.

b) (8 points) Establish an equation in one variable for the area of the body of the carpet.

c) (12 points) Find the carpet with the maximum possible area in the body. Be sure to show why your answer is a maximum.

5) Evaluate the following integrals.

a) (6 points) $\displaystyle\int_0^{\pi} \cos(x) - 21x^2 \, dx$

b) (8 points) $\displaystyle\int \frac{3x^2 - 1}{(x^3 - x + 22)^{4/9}} \, dx$

c) (11 points) $\displaystyle\int_0^{\pi/4} \sqrt{1 - \tan^2(x)} \sec^2(x) \, dx$

BONUS: (10 points) For all everywhere-differentiable f such that $f(1) = 5$ and $f(4) = 13$, compute $\displaystyle\int_{-1}^{2} \frac{d}{dx}(f(x^2))\, dx$.