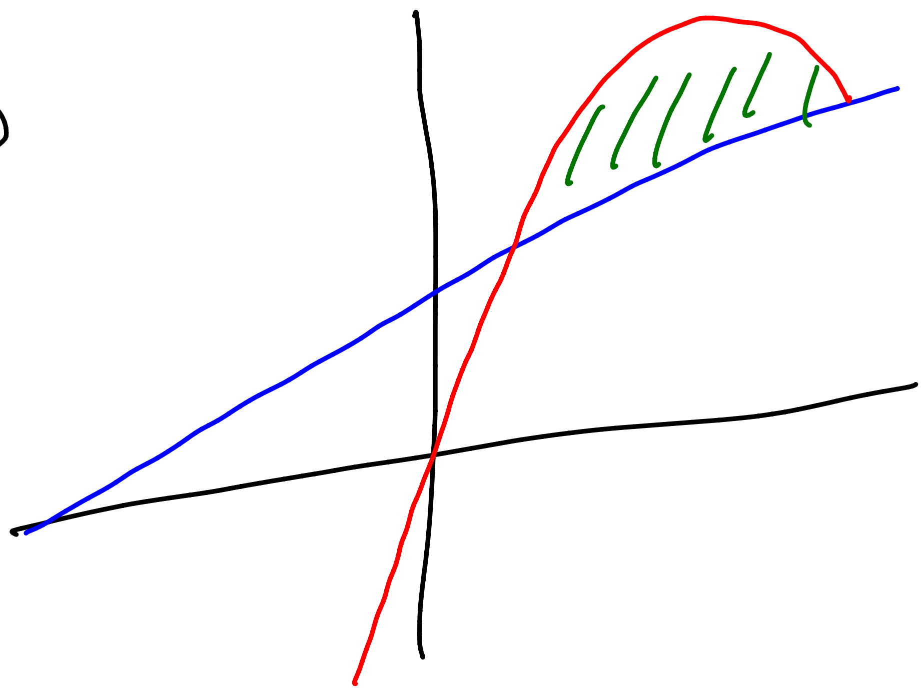
Fall 2011 Exam 3

a)



b) $-x^2+8x = 2x+5$

$x^2-6x+5=0$

$(x-5)(x-1)=0$

$x=1, 5$

c) $$\int_{-1}^{1} (2x+5)-(-x^2+8x)\,dx$$

$$+\int_{1}^{3} (-x^2+8x)-(2x+5)\,dx$$

$$=\int_{-1}^{1} x^2-6x+5\ \ dx$$

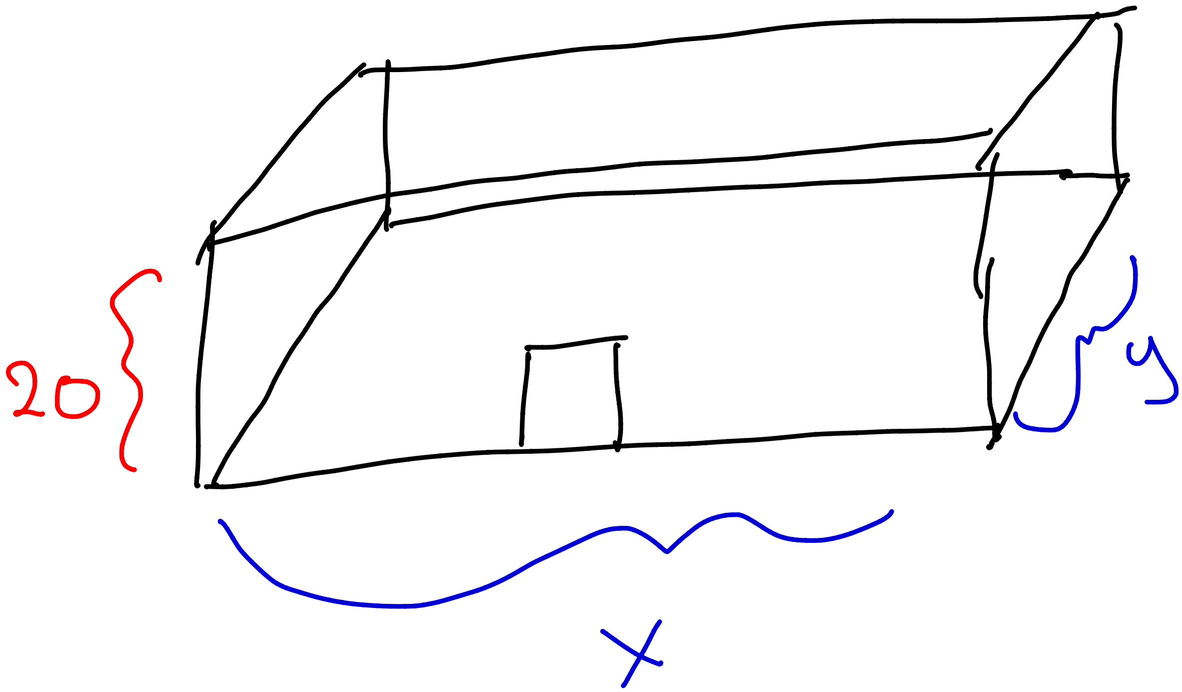
$$-\int_{1}^{3} x^2-6x+5\ \ dx$$

$$=\left(\frac{x^3}{3}-3x^2+5x\right)\Big|_{-1}^{1}$$

$$-\left(\frac{x^3}{3}-3x^2+5x\right)\Big|_{1}^{3}$$

$$=\left(2\tfrac{1}{3}+8\tfrac{1}{3}\right)-\left(-3-2\tfrac{1}{3}\right)=16$$

2) a)



b) $C = 14(20x) + 2\cdot 10(20y) + 10\cdot(20x)$

$=$

$A = xy = 14{,}520$

$$y = \frac{14{,}520}{x}$$

$$C(x) = 40\left(12x + \frac{145{,}200}{x}\right)$$

$$C'(x) = 40\left(12 - 145{,}200\, x^{-2}\right)$$

$$0 = 40\left(12 - 145{,}200\, x^{-2}\right)$$

$$\frac{12}{145{,}200} = \frac{1}{x^2}$$

$$x = \sqrt{\frac{145{,}200}{12}} = \sqrt{12100} = 110$$

$$C''(x) = 40\left(290{,}400\, x^{-3}\right)$$

$$C''(110) > 0$$

so we have a min

$x = 110$ ft

$$y = \frac{14520}{110} = 132 \text{ ft}$$

Amount $= 2 \cdot (20x) + 2(20y)$

$= 40\,(110 + 132) = 9680 \text{ ft}^2$

c) $\displaystyle\int_0^{\pi/4} \sqrt{1-\tan(x)}\ \sec^4(x)\,dx$

$$= \int_0^{\pi/4} \sqrt{1-\tan(x)}\ \sec^2(x)\sec^2(x)\,dx$$

$$= \int_0^{\pi/4} \sqrt{1-\tan(x)}\ \left(\tan^2(x)+1\right)\left(\sec^2(x)\right)dx$$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$u(0) = 0$

$u(\pi/4) = 1$

$$= \int_0^1 \sqrt{1-u}\ (u^2+1)\,du$$

$w = 1-u \quad , \quad u = 1-w$

$dw = -1\,du \qquad w(0) = 1$

$w(1) = 0$

$$-\int_1^0 \sqrt{w}\left((1-w)^2+1\right)dw$$

$$= \int_0^1 \sqrt{w}\left(w^2-2w+2\right)dw$$

$$= \int_0^1 w^{5/2}-2w^{3/2}+2w^{1/2}\,dw$$

$$= \left(\frac{2w^{7/2}}{7}-\frac{4w^{5/2}}{5}+\frac{4w^{3/2}}{3}\right)\Bigg|_0^1$$

$$= 2/7 - 4/5 + 4/3$$

$$= \frac{30}{105}-\frac{84}{105}+\frac{140}{105}$$

$$= \frac{86}{105}$$

4)

recall:

Volume of a sphere $= 4/3 \pi r^3$

$V = r+2$, so

$4/3 \pi r^3 = r+2$ is the same as

$4/3 \pi r^3 - r - 2 = 0$

Let $f(r) = 4/3 \pi r^3 - r - 2$

$f(0) = -2 < 0$

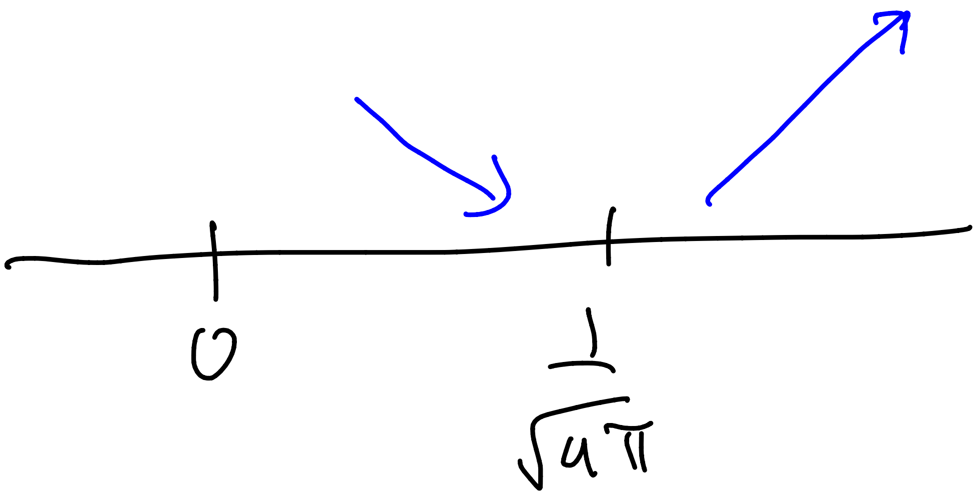
$f(1) = \frac{4\pi}{3} - 3 > 0$

By the intermediate value theorem, f has a zero, so it is possible!

a) $f'(r) = 4\pi r^2 - 1$

$0 = f'(r) = 4\pi r^2 - 1$

$r = \frac{1}{\sqrt{4\pi}}$



f has a local min at $r = \frac{1}{\sqrt{4\pi}}$

and $f(0) < 0$, so f decreases

until $r = \frac{1}{\sqrt{4\pi}}$, and $f\left(\frac{1}{\sqrt{4\pi}}\right) < 0$

Then there is only one value of

r where f equals zero, somewhere

after $r = \frac{1}{\sqrt{4\pi}}$ and before $r = 1$.

b) $$r_2 = r_1 - \frac{f(r_1)}{f'(r_1)}$$

$$= 1 - \frac{\frac{4\pi}{3} - 3}{4\pi - 1}$$

$$r_3 = r_2 - \frac{f(r_2)}{f'(r_2)}$$

$$= 1 - \frac{\frac{4\pi}{3} - 3}{4\pi - 1}\left(\frac{\frac{4\pi}{3}\left(1 - \frac{\frac{4\pi}{3} - 3}{4\pi - 1}\right)^3 - \left(1 - \frac{\frac{4\pi}{3} - 3}{4\pi - 1}\right) - 2}{4\pi\left(1 - \frac{\frac{4\pi}{3} - 3}{4\pi - 1}\right)^2 - 1}\right)$$

$$\approx .883157$$