Fall '22 Exam 1

a) On the yz-plane, $x=0$, so

we get $-2y+3z=8$.

For the y-intercept, $z=0$,

so $-2y=8$, $y=-4$.

The point is $(0,-4,0)$.

b) If the base is 4 and the hypotenuse is 5, then

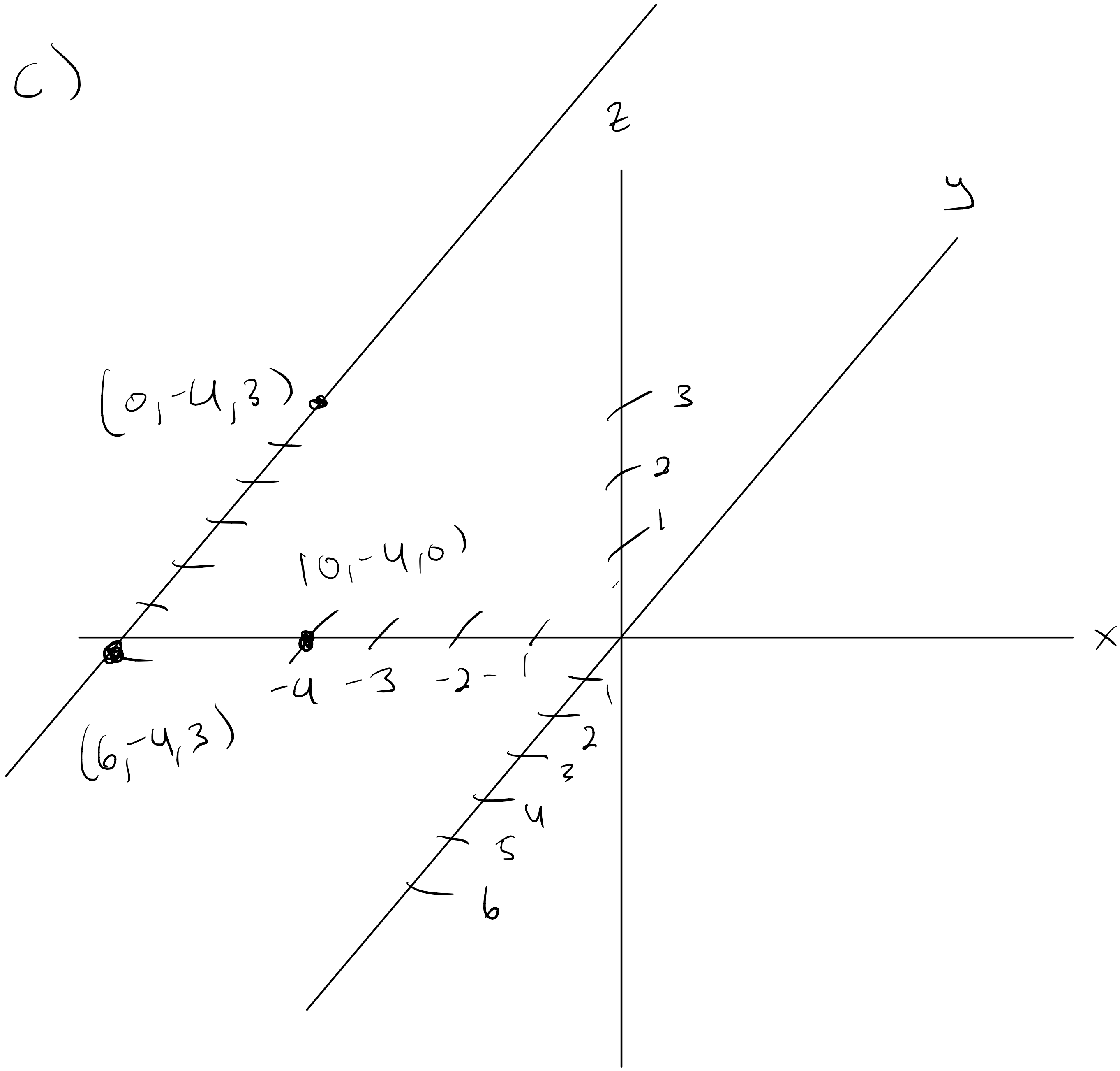
$$h^2+4^2=5^2$$

$$h^2=9$$

$$h=3$$

The point is $(0, -4, 3)$

c)



2) For the domain, we need

$$\frac{x^2}{4} + \frac{y^2}{9} - 1 > 0 \quad {+1}$$

a) For $(5, 3)$,

$$\frac{5^2}{4} + \frac{3^2}{9} - 1$$

$$= \frac{25}{4} > 0$$

So $(5, 3)$ is in the domain.

b) For $(-1, 1)$,

$$\frac{(-1)^2}{4} + \frac{1^2}{9} - 1$$

$$= 1/4 + 1/9 - 1$$

$$= -23/36 < 0,$$ so not in the domain.

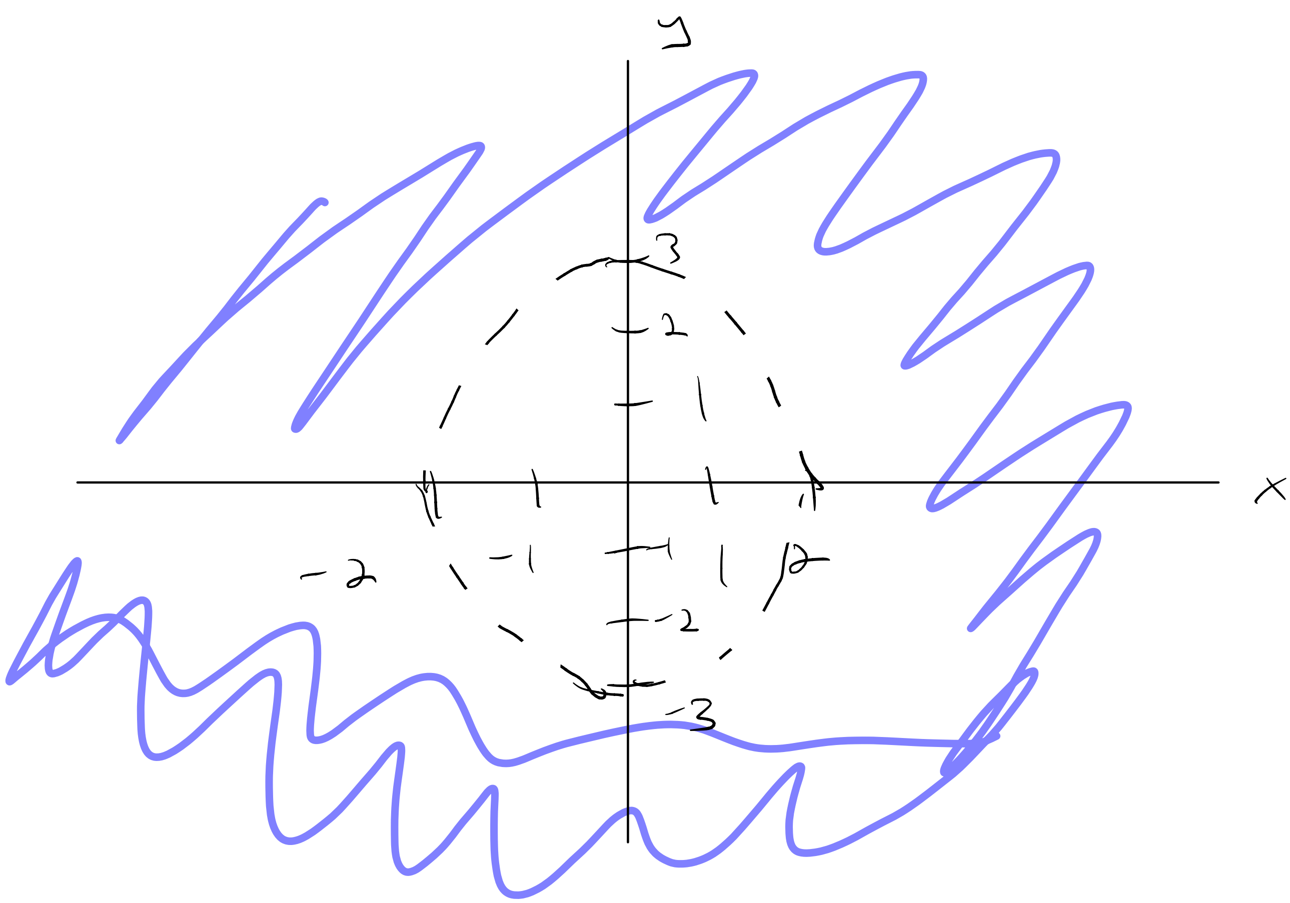
c) Draw $\frac{x^2}{4} + \frac{y^2}{9} - 1 = 0.$

If $x = 0$, $\frac{y^2}{9} - 1 = 0$

$y = \pm 3$

If $y = 0$, $\frac{x^2}{4} - 1 = 0$

$x = \pm 2$



3) If $y = \dfrac{2x-1}{3x+4}$, then

$$(3x+4)y = 2x-1$$

$$3xy + 4y = 2x-1$$

$$3xy - 2x = -1-4y$$

$$x(3y-2) = -1-4y$$

$$x = \frac{-1-4y}{3y-2}$$

The only issue is when

$3y-2=0$, that is,

when $y = 2/3$.

So all real numbers y with

$y \neq 2/3$ is the range

Straight-up plug in

$$-A + C = D$$

$$3B + 2C = D$$

$$8A + 3B + 6C = D$$

$$-A + C = 3B + 2C \text{, so}$$

$$3B = -A - C$$

$$3B + 2C = 8A + 3B + 6C$$

$$-A - C + 2C = 8A - A - C + 6C$$

$$-A + C = 7A + 5C$$

$$8A = -4C$$

$$C = -2A$$

so

$$3B = -A - C$$

$$= -A + 2A$$

$$= A$$

$$B = \frac{A}{3}$$

$$D = -A + C = -A + (-2A) = -3A$$

so we get

$$Ax + \frac{A}{3}y - 2Az = -3A$$

and if $A = 1$,

$$x + \frac{y}{3} - 2z = -3$$

5) a)

$$m = \frac{g(2+h) - g(2)}{2+h-2} = \frac{g(2+h) - g(2)}{h}$$

$$= \frac{\frac{1}{\sqrt{9-(2+h)^3}} - 1}{h}$$

b) $$\frac{\frac{1}{\sqrt{9-(4+4h+h^2)(2+h)}} - 1}{h}$$

$$= \frac{\frac{1}{\sqrt{9-(h^3+6h^2+12h+8)}} - 1}{h}$$

$$= \frac{\frac{1}{\sqrt{1-h^3-6h^2-12h}} - 1}{h}$$

$$\frac{\dfrac{1}{\sqrt{1-h^3-6h^2-12h}}-1}{h}\cdot\frac{\dfrac{1}{\sqrt{1-h^3-6h^2-12h}}+1}{\dfrac{1}{\sqrt{1-h^3-6h^2-12h}}+1}$$

$$=\frac{\dfrac{1}{(1-h^3-6h^2-12h)}-1}{h\left(\dfrac{1}{\sqrt{1-h^3-6h^2-12h}}+1\right)}$$

$$=\frac{\dfrac{1-(1-h^3-6h^2-12h)}{(1-h^3-6h^2-12h)}}{h\left(\dfrac{1}{\sqrt{1-h^3-6h^2-12h}}+1\right)}$$

$$= \frac{\left(h^3+6h^2+12h\right)\cdot\frac{1}{1-h^3-6h^2-12h}}{h\left(\frac{1}{\sqrt{1-h^3-6h^2-12h}}+1\right)}$$

$$= \frac{\cancel{h}\left(h^2+6h+12\right)\cdot\frac{1}{1-h^3-6h^2-12h}}{\cancel{h}\left(\frac{1}{\sqrt{1-h^3-6h^2-12h}}+1\right)}$$

Now let $h=0$

$$= \frac{12}{1}\cdot\frac{1}{2} = 6$$

$$y - 1 = 6(x-2)$$