Exam 2 Fall '22

1) a)

$$-\sin(x)\,(8x^5 - 4x^2 + 9x + e^2)$$

$$+\cos(x)\,(40x^4 - 8x + 9)$$

b) $$\frac{(17x^3 - 12x^2)\,e^x - e^x\,(51x^2 - 24x)}{(17x^3 - 12x^2)^2}$$

c) $h(x) = \sin\left(\sqrt{4\ln(x)}\right)$

$= \sin\left(2\sqrt{\ln(x)}\right)$

$= \sin\left(2\,(\ln(x))^{1/2}\right)$

$h'(x) = \cos\left(2(\ln(x))^{1/2}\right)$

$\cdot\, (\ln(x))^{-1/2} \cdot \frac{1}{x}$

2) $f(x,y) = \tan(\pi x^2 y^6) - \ln(xy^2)$

$$= \tan(\pi x^2 y^6) - \ln(x) - \ln(y^2)$$

$+1$

$$= \tan(\pi x^2 y^6) - \ln(x) - 2\ln(y)$$

$+1$

$$\frac{\partial f}{\partial x} = \sec^2(\pi x^2 y^6) \cdot 2\pi x y^6 - \frac{1}{x}$$

$$\frac{\partial f}{\partial x}(4, 1/2) = 4\pi \cdot 4 \cdot \frac{1}{64} - \frac{1}{4} = \frac{\pi - 1}{4}$$

$$\frac{\partial f}{\partial y} = \sec^2(\pi x^2 y) \cdot 6\pi x^2 y^5 - \frac{2}{y}$$

$$\frac{\partial f}{\partial y}(4, 1/2) = 12\pi \cdot \frac{16}{32} - 4$$

$$= 6\pi - 4$$

$$m = \frac{-\frac{\partial f}{\partial x}(4, 1/2)}{\frac{\partial f}{\partial y}(4, 1/2)} = -\left(\frac{\pi - 1}{4(6\pi - 4)}\right)$$

$$\tan(\pi x^2 y^6) - \ln(xy^2)$$

$$= \tan(\pi x^2 y^6) - \ln(x) - \ln(y^2)$$

$$= \tan(\pi x^2 y^6) - \ln(x) - 2\ln(y)$$

$$0 = \sec^2(\pi x^2 y^6) \cdot \left(2\pi x y^6 + 6y^5 \frac{dy}{dx} \cdot \pi x^2\right)$$

$$- \frac{1}{x} - \frac{2}{y}\frac{dy}{dx}$$

$$= \left(\sec^2(\pi x^2 y^6) \cdot 2\pi x y^6 - \frac{1}{x}\right)$$

$$+ \frac{dy}{dx}\left(\sec^2(\pi x^2 y^6) 6y^5 \pi x^2 - \frac{2}{y}\right)$$

$$\frac{dy}{dx} = \frac{\frac{1}{x} - \sec^2(\pi x^2 y^6) \cdot 2\pi x y^6}{\sec^2(\pi x^2 y^6) 6y^5 \pi x^2 - 2/y}$$

$$@ (4, 1/2): \quad \frac{\frac{1}{4} - \frac{\pi}{4}}{6\pi - 4} = \frac{dy}{dx}$$

$$-\left(\frac{\pi - 1}{4(6\pi - 8)}\right)(x-4) = y - \tfrac{1}{2}$$

$$\frac{\frac{1}{8} - \frac{1}{4}}{\frac{3}{2} - 4} = \frac{-\frac{1}{8}}{-\frac{5}{2}}$$

3) $\frac{\partial f}{\partial x} = e^{x^2 y - y^3 x} \cdot (2xy - y^3)$

$\frac{\partial f}{\partial x}(4,2) = 16 - 8 = 8$

$\frac{\partial f}{\partial y} = e^{x^2 y - y x} \cdot (x^2 - 3y^2 x)$

$\frac{\partial f}{\partial x}(4,2) = 16 - 3 \cdot 16 = -32$

$8(x-4) - 32(y-2) - (z-1) = 0$

-or-

$8x - 32y - z = D$

plug in

$32 - 64 - 1 = D$

$D = -33$

4) a) $1-\cos^2(3x) = \sin^2(3x)$, so

$$\lim_{x\to 0} \frac{\sin^2(3x)}{16x^2} = \lim_{x\to 0} \frac{\sin(3x)}{4x} \cdot \frac{\sin(3x)}{4x}$$

$$= \frac{1}{16} \cdot \lim_{x\to 0} \frac{\sin(3x)}{x} \cdot \frac{\sin(3x)}{x}$$

$$= \frac{9}{16} \cdot \lim_{x\to 0} \frac{\sin(3x)}{3x} \cdot \frac{\sin(3x)}{3x}$$

$$= \frac{9}{16} \cdot 1 \cdot 1 = 9/16$$

b) $$\lim_{x\to 4} \frac{\ln\left(\tan^2\left(\frac{4\pi}{3x}\right)\right) - \ln(3)}{2x^2-3x-20}$$

$$= \lim_{x\to 4} \frac{\ln\left(\tan^2\left(\frac{4\pi}{3x}\right)\right) - \ln(3)}{(2x+5)(x-4)}$$

$$= \lim_{x\to 4} \frac{1}{2x+5} \cdot \frac{\ln\left(\tan^2(4\pi/3x)\right) - \ln(3)}{x-4}$$

Plugging in $x = 4$:

$$\ln\left(\tan^2\left(\frac{4\pi}{4\cdot 3}\right)\right) - \ln(3)$$

$$= \ln\left(\tan^2(\pi/3)\right) - \ln(3)$$

$$= \ln(3) - \ln(3)$$

So

$$= 0$$

$$\lim_{x \to 4} \frac{\ln\left(\tan^2\left(\frac{4\pi}{3x}\right)\right) - \ln(3)}{x - 4}$$

$$= f'(4) \quad \text{where} \quad f(x) = \ln\left(\tan^2\left(\frac{4\pi}{3x}\right)\right)$$

$$f'(x) = \frac{1}{\tan^2\left(\frac{4\pi}{3x}\right)} \cdot 2\tan\left(\frac{4\pi}{3x}\right) \cdot \sec^2\left(\frac{4\pi}{3x}\right) \cdot \frac{-4\pi}{3x^2}$$

$$f'(4) = \frac{1}{3} \cdot 2 \cdot \sqrt{3} \cdot 4 \cdot \left(\frac{-4\pi}{3 \cdot 4^2}\right)$$

$$= \frac{-2\pi}{3\sqrt{3}}$$

Since $\lim_{x \to 4} \frac{1}{2x+5} = \frac{1}{13}$, we get

$$\frac{1}{13} \cdot \frac{-2\sqrt{4}}{3\sqrt{3}} = \frac{-2\sqrt{4}}{39\sqrt{3}}$$