Exam 3 Fall '22

1) $f'(x) = \cos(x) - x\sin(x) - \cos(x)$

$= -x\sin(x)$

$= 0$

$x = 0$ or $\sin(x) = 0$

$x = 0, \pi, -\pi$

$(-4 \leq x \leq 5)$

$x = -4, 5$

$f(0) = 0$ $\quad$ $f(\pi) = -\pi$

$f(-\pi) = \pi$

$f(-4) = -4\cos(4) + \sin(4)$

$f(5) = 5\cos(5) - \sin(5)$

min: $-\tilde{d}$

max: $\tilde{h}$

2) a) $\frac{\partial g}{\partial x} = 6y - 3 = 0, \quad y = 1/2$

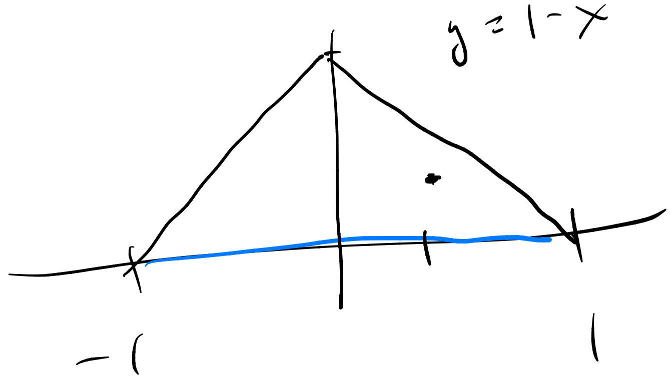
$\frac{\partial g}{\partial y} = 6x - 2 = 0, \quad x = 1/3$

$(1/3, 1/2)$

$y = 1 - x$, when $x = 1/3$,

$y = 2/3 > 1/2$

b)



yes

c) Yes - plug the boundary curves into the equation for g and use 1-variable calculus to find maxima & minima on the boundary. Then plug all point into g, biggest value is max, smallest is min

3) a)

$$y = \sqrt{36 - x^2}$$

$$y^2 = 36 - x^2$$

$$x^2 + y^2 = 36$$

circle of radius 6

$$\int_{-6}^{6} \sqrt{36 - x^2}\, dx$$

$$= \frac{1}{2}\left(\text{area of a circle of radius } 6\right)$$

$$= \frac{1}{2} \cdot 36\pi = 18\pi$$

b) $u = e^x, \quad du = e^x\,dx$

$$\int \cos(u)\,du$$

$$= \sin(u) + C$$

$$= \sin(e^x) + C$$

c) Suppose $g'(t) = \ln(\sqrt{t} + 1)$.

Then $\displaystyle\int_1^{x^7} \ln(\sqrt{t}+1)\,dt = g(x^7) - g(1)$

$$\frac{d}{dx}\left(\int_1^{x^4} \ln(\sqrt{t}+1)\,dt\right)$$

$$= \frac{d}{dx}\left(g(x^7) - g(1)\right)$$

$$= 7x^6 g'(x^7) = 7x^6 \ln\left(x^{7/2} + 1\right)$$