Final '22

1) a) T

b) T

c) T

d) F

e) F

2) a) slope, tangent

b) area, x

c) 1

3)

a) $x \cdot \frac{1}{x} + \ln(x)$

$= 1 + \ln(x)$

b) $$\frac{e^{x} \cdot \cos(x) - \sin(x) e^{x}}{e^{2x}}$$

c) $\sec^2(x^7 + \sec(x)) \cdot (7x^6 + \sec(x)\tan(x))$

4)

$$f'(x) = -\sin(\ln(x+1)) \cdot \frac{1}{x+1} \cdot e^x$$

$$+ \cos(\ln(x+1)) \cdot e^x$$

$$m = f'(0) = 1$$

$$f(0) = 1$$

$$y - 1 = x$$

$$y = x+1$$

5) a) $g'(x) = 6x^2 - 12x - 18$

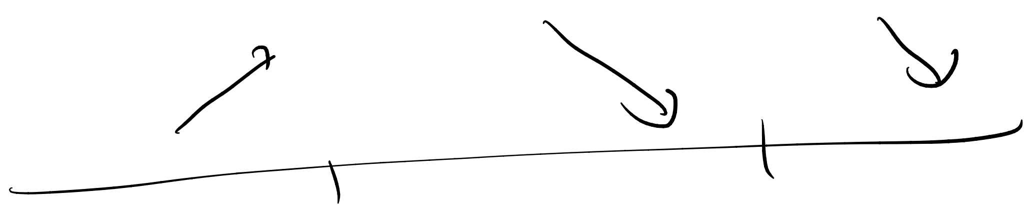
$= 6(x^2 - 2x - 3)$

$= 6(x-3)(x+1)$

$= 0$

$x = 3, \quad x = -1$

b)



-1 $\quad$ 3 $\quad$ $g(4) > 0$

$g(-2) > 0$ $\quad$ $g(0) = -18$

increasing $(-\infty, -1), \ (3, \infty)$

decreasing $(-1, 3)$

c) local max: $x = -1$

local min: $x = 3$

d) $g''(x) = 12x - 12$

$= 0$

$x = 1$

$g''(0) = -12$ $\quad$ $g''(2) = 12$

concave up $(1, \infty)$

concave down $(-\infty, 1)$

e) $x = 1$

6)

$$\frac{\partial f}{\partial x} = -2\cos(xy)\sin(xy)\cdot y$$

$$+2\cos(xy)\sin(xy)\cdot y$$

$$=0$$

$$\frac{\partial f}{\partial y} = -2\cos(xy)\sin(xy)\cdot x$$

$$+2\cos(xy)\sin(xy)\cdot x$$

$$=0$$

$A=0$, $B=0$

$-z = D$

$D=-1$

$z=1$

- or -

$z = 1$ since $\cos^2(xy) + \sin^2(xy) = 1$

7) $f(x,y) = e^{x^2-3xy} + y^2$

$$\frac{\partial f}{\partial x} = e^{x^2-3xy} \cdot (2x-3y)$$

$$\frac{\partial f}{\partial y} = e^{x^2-3xy} \cdot (-3x) + 2y$$

$$\frac{\partial f}{\partial x}(3,1) = 3$$

$$\frac{\partial f}{\partial y}(3,1) = -7$$

$$m = \frac{3}{7}$$

$$y-1 = \frac{3}{7}(x-3)$$

8) a) $\frac{\partial g}{\partial x} = 4y - 12 = 0, \; y = 3$

$\frac{\partial g}{\partial y} = 4x + 16 = 0, \; x = -4$

$(-4, 3)$

b) $4^2 + 3^2 = 25 < 49$

so $(-4, 3)$ is in D.

c) same as exam 3

a) a) $\displaystyle\int_{-\pi/2}^{0} \frac{\sin(x)}{e^{\cos(x)}}\,dx$

$\displaystyle = \int_{-\pi/2}^{0} \sin(x)\, e^{-\cos(x)}\,dx$

$u = -\cos(x)$

$du = \sin(x)\,dx$

$u(-\pi/2) = 0$

$u(0) = -1$

$\displaystyle\int_{0}^{-1} e^{u}\,du = e^{u}\Big|_{0}^{-1} = \frac{1}{e} - 1$

b) $y = \sqrt{49 - x^2}$

$y^2 = 49 - x^2$

$x^2 + y^2 = 49$

circle of radius 7

$\int_{-7}^{0} \sqrt{49 - x^2}\, dx = 1/4$ (area of circle)

$= \frac{49\pi}{4}$

10) a) $\frac{3e^0}{81-36} = \frac{3}{45} = \frac{1}{15}$

b) $\lim_{x \to 0} \sin^2(x) - x = 0$

$\lim_{x \to 0} \tan(x) + 7x = 0$

l'Hopital

$\lim_{x \to 0} \frac{\sin^2(x) - x}{\tan(x) + 7x}$

$= \lim_{x \to 0} \frac{2\sin(x)\cos(x) - 1}{\sec^2(x) + 7}$

$= -\frac{1}{8}$

c) $\left(\frac{x}{3+x}\right)^{14x}$

$$= \left(\frac{3+x}{x}\right)^{-14x}$$

$$= \left(1+\frac{3}{x}\right)^{-14x}$$

$$= e^{\ln\left(1+\frac{3}{x}\right)^{-14x}}$$

$$= e^{-14x\ln\left(1+\frac{3}{x}\right)}$$

$$\lim_{x\to\infty} -14x\ln\left(1+\frac{3}{x}\right)$$

$$= -14\lim_{x\to\infty}\frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{x}}$$

$$= -14\lim_{x\to\infty}\frac{\frac{1}{1+\frac{3}{x}}\cdot 3\cdot \cancel{-\frac{1}{x^2}}}{\cancel{-\frac{1}{x^2}}}$$

$= -42$

final e^{-42}

11 a) $xy = A$

b) $\left(\frac{x}{2}\right)^2 + y^2 = 43^2 = 1849$

$$\frac{x^2}{4} + y^2 = 1849$$

$$y^2 = 1849 - \frac{x^3}{4}$$

$$y = \sqrt{1849 - \frac{x^2}{4}}$$

$$A = x \cdot \sqrt{1849 - \frac{x^2}{4}}$$

$$= \frac{x}{2} \cdot \sqrt{7396 - x^2}$$

c) $\frac{x}{2} \cdot \sqrt{7396 - x^2}$

$A'(x) = \frac{1}{2} \cdot \sqrt{7396 - x^2}$

$+ \frac{x}{2} \cdot \frac{1}{2} \cdot (7396 - x^2)^{-1/2} \cdot (-2x)$

$A'(x) = \frac{1}{2} \left((7396 - x^2)^{-1/2} \right.$

$\left. \cdot (7396 - x^2 - x^2) \right)$

$A'(x) = (7396 - x^2)^{-1/2} \cdot (3698 - x^2)$

$0 = A'(x)$

$0 = 3698 - x^2$

$x^2 = 3698$

$x = 43\sqrt{2}$

$$y = \sqrt{1849 - \frac{3698}{4}}$$

$$y = \sqrt{\frac{1849}{2}}$$

$$= \frac{43}{\sqrt{2}}$$

$$43\sqrt{2} \times \frac{43}{\sqrt{2}}$$