Name:

Math 115 Final

December 21st, 2011

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) Compute the derivatives of the following functions.

a) (4 points) $f(x) = 15x^7 \tan(x)$

b) (6 points) $g(x) = \dfrac{4x - 8}{12x + 3}$

c) (8 points) $h(x) = \cos^2(24x)$

2) (8 points) Find the equation of the tangent line to the graph of

$$f(x) = 6x^5 - 18x^3 - 45$$

at the point $(2, 3)$.

3) Consider the function $f(x) = 2x^4 - 12x^2 + 92$.

a) (10 points) Find all local maxima, local minima, and intervals of increase/decrease for f.

b) (10 points) Determine the intervals of concavity and inflection points (if any exist) for f.

4) Evaluate the following integrals.

a) (5 points) $\int (-42x^5 + 6x^7)\, dx$

b) (7 points) $\int_{-\pi/6}^{\pi/6} \frac{\sin(x)}{\sqrt{16 - 14\cos(x)}}\, dx$

c) (11 points) $\int_0^1 x^4\sqrt{1 - x^{10}}\, dx$

5) Find the value of the limits, if they exist.

a) (4 points) $\lim_{x \to 9} \dfrac{\sqrt{13 - x} - 2}{x - 7}$

b) (9 points) $\lim_{x \to -4} \dfrac{|x^2 + 4x|}{x + 4}$

c) (11 points) $\lim_{x \to 0} \dfrac{\tan^2(8x)}{x^2}$

6) a) (3 points) Define what it means for a function f to be continuous at a point $x = a$.

b) (11 points) Find all values of k (if any exist) that make the function

$$f(x) = \begin{cases} kx^2 - 4x & x > 3 \\ x - 6kx & x < 3 \\ 8 & x = 3 \end{cases}$$

continuous at $x = 3$. Be sure to show your work and be sure that your work is in accord with the definition provided in part a)!

7) Let $f(x) = x^{11} + 14x^7 + 8x - 19$.

a) (5 points) Show that f has a real zero.

b) (9 points) Show that f has only one such zero.

c) (7 points) Starting with $x_1 = 1$, apply Newton's method to find x_3. You may leave your answer in unsimplified form.

8) A paper cup has the shape of a right circular cone with a radius of 6 cm at the top and a height of 24 cm. Water pours into the cup at a rate of 3 cubic centimeters per second. Note that the volume of such a cone is $\frac{1}{3}\pi r^2 h$.

a) (5 points) Draw a picture representing the above scenario, labeling your variables.

b) (7 points) Find an equation in one variable for the volume of the water in the cup. (*Hint*: similarity)

c) (7 points) How fast is the water level rising when the water in the cup is 4 cm high?

9) Consider the region bounded by $y = \sqrt{x-6}$ by $y = (x-6)^{2/3}$ from $x = 6$ to $x = 7$.

a) (3 points) Draw a picture of the region.

b) (5 points) Find an integral representing the volume obtained by revolving the region about the x-axis

c) (7 points) Compute the volume of the solid obtained in part b).

10) A rectangular poster with a picture of Sasquatch on it is to have an area of 154 square inches with one-inch margins at the bottom and sides and two-inch margins at the top. Sasquatch's picture must be inside the margins.

a) (4 points) Draw a picture that reflects this scenario.

b) (7 points) Establish an equation in one variable for the area of the poster on which you can put a picture of Sasquatch.

c) (7 points) Find the dimensions of the poster with the most area for a picture of Sasquatch satisfying the given conditions.