Math 115 Final

December 18th, 2013

1) Compute the derivatives of the following functions.

a) (4 points) $f(x) = 14x^3 \sin(x)$

b) (6 points) $g(x) = \dfrac{8 - 5x}{15x + 9}$

c) (8 points) $h(x) = \tan^2(11x)$

2) (11 points) Find the equation of the tangent line to the graph of

$$\cos(y) - x^2 y + \pi/2 = 0$$

at the point $(1, \pi/2)$.

3) Consider the function $f(x) = x(x-6)^3$.

a) (10 points) Find all local maxima, local minima, and intervals of increase/decrease for f.

b) (10 points) Determine the intervals of concavity and inflection points (if any exist) for f.

4) Evaluate the following integrals.

a) (5 points) $\int \left(-5x^2 + 10x^9\right)\ dx$

b) (7 points) $\int_0^2 \frac{x^3}{\sqrt{x^4+1}}\ dx$

c) (11 points) $\int_{\pi/6}^{\pi/2} \cos^3(x)\sin^2(x)\ dx$

5) Find the value of the limits, if they exist.

a) (4 points) $\lim_{x \to \pi/4} \dfrac{\sin(x)}{10x}$

b) (9 points) $\lim_{x \to 0} \dfrac{\tan(x) + 3x}{7x}$

c) (11 points) $\lim_{x \to \infty} (\sqrt{x+1} - \sqrt{x+3})$

6) a) (3 points) Define what it means for a function f to be continuous at a point $x = a$.

b) (10 points) Find all values of k (if any exist) that make the function

$$f(x) = \begin{cases} k^2 + kx & x > 5 \\ \frac{20k}{x} + 6 & x < 5 \\ -6 & x = 5 \end{cases}$$

continuous at $x = 5$. Be sure to show your work and be sure that your work is in accord with the definition provided in part a)!

7) Let $f(x) = 6x^9 + 12x^5 + 3x - 24$.

a) (5 points) Show that f has a real zero (root).

b) (6 points) Show that f has only one such zero (root).

c) (7 points) Starting with $x_1 = 1$, apply Newton's method to find x_3. You may leave your answer in unsimplified form.

8) Consider the region $\mathcal{R}$ bounded by the curves $y = 2x - x^2 + 3$ and $y = 3 - x$.

a) (5 points) Find the intersection points of the two curves.

b) (4 points) Set up an integral representing the area of $\mathcal{R}$ BUT DO NOT EVALUATE THE INTEGRAL.

c) (6 points) Set up an integral for the volume obtained by revolving $\mathcal{R}$ about the x-axis BUT DO NOT EVALUATE THE INTEGRAL.

9) Sasquatch's favorite drinking cup has the shape of a cylinder with no top. Sasquatch is planning to go into business by mass-marketing such cups with his partner, Bigfoot. He wants the cups to contain a fixed volume of 9000π cm^3. The insulated material for the sides costs more, at .03 cents per cm^2, than the base, which costs .01 cents per cm^2. The formula for the surface area of a cylinder with no top of radius r and height h is $\pi r^2 + 2\pi rh$ where πr^2 is the area of the base and $2\pi rh$ is the area of the sides.

a) (5 points) Establish an equation in one variable for the cost of Sasquatch's cup.

b) (12 points) Find the height and radius of a cup that minimizes the cost for Sasquatch to produce. Be sure to show your answer is actually a minimum.

10) A basketball court is 50 feet from side-to-side and 94 feet from end-to-end. Vice-President Joe Biden is sitting 10 feet away from one of the sides level with half-court watching President Barack Obama fast-break from end-to-end down the dead-center of the court. President Obama runs at a rate of 7 ft/s and starts at one end of the court.

a) (5 points) Draw a picture representing the above scenario, labeling your variables.

b) (5 points) Find an equation in relating the angle between the president and the vice-president to the distance between them.

c) (11 points) How fast is the angle between the president and the vice-president changing 5 seconds after President Obama begins running?