Name:

Math 115 Final

December 13, 2022

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless indicated, DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.
4. If you have a question, raise your hand or come up and ask me.

1) (10 points, 2 points each) True/False. No justification is necessary.

a) If f has a local maximum or a local minimum at $x = c$, then $f'(c) = 0$ or $f'(c)$ does not exist.

b) Every continuous function on a finite interval (a, b) has an antiderivative.

c) If f is differentiable at $x = 2$, then f is continuous at $x = 2$.

d) For all differentiable functions f and g,

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g'(x)$$

e) For ALL real numbers x and y, $\sqrt{x^2 - y^2} = x - y$.

2) (10 points, 2 points each) Fill-in-the-blank. No justification is necessary.

a) The derivative is the ____________________ of the ___________________________ line.

b) The integral $\int_{-7}^{0} \sqrt{49 - x^2}\, dx =$ represents the ______________ between the graph of the function $y = \sqrt{49 - x^2}$ and the _______-axis from $x = 0$ to $x = 7$.

c) $\sin^2(\theta) + \cos^2(\theta) =$ ______________.

3) Compute the derivatives of the following functions.

a) $f(x) = x\ln(x)$

b) $g(x) = \dfrac{\sin(x)}{e^x}$

c) $h(x) = \tan(x^7 + \sec(x))$

4) Find the equation of the tangent line to the function

$$f(x) = \cos(\ln(x+1)) \cdot e^x$$

at the point $x = 0$.

5) Let $g(x) = 2x^3 - 6x^2 - 18x + 5$.

a) Find all critical numbers for g, if any exist.

b) Determine the intervals where g is increasing or where g is decreasing.

c) Record the x-coordinates of the local maxima and minima for g, if any exist.

d) Determine the intervals of concavity for g.

e) Find all inflection points for g, if any exist.

6) Compute the equation of the tangent plane to the graph of

$$z = f(x, y) = \cos^2(xy) + \sin^2(xy)$$

at the point $(-2, \pi/4, 1)$.

7) Find the equation of the tangent line to the graph of

$$2 = e^{x^2 - 3xy} + y^2$$

at the point $(3, 1, 2)$.

8) Let $g(x, y) = 4xy - 12x + 16y - 1$.

a) Find all critical points of g.

b) Let C be the circle in the xy-plane determined by the equation

$$x^2 + y^2 = 49$$

and let $\mathcal{D}$ be the region in the xy-plane consisting of all points either inside or on C. Are any of the points you found in a) in the region $\mathcal{D}$?

c) Does g have an absolute minimum on $\mathcal{D}$? If not, explain why not, and if so, describe how you would find the minimum of g on $\mathcal{D}$ WITHOUT doing any calculations.

9) Evaluate the following integrals.

a) $\displaystyle\int_{-\pi/2}^{0} \frac{\sin(x)}{e^{\cos(x)}}\ dx$

b) $\displaystyle\int_{-7}^{0} \sqrt{49 - x^2}\ dx$

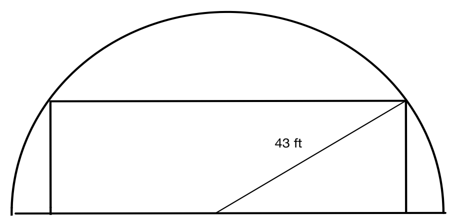
10) Find the value of the limits, if they exist.

a) $\lim\limits_{x\to 9} \dfrac{3e^{x-9}}{x^2 - 4x}$

b) $\lim\limits_{x\to 0} \dfrac{\sin^2(x) - x}{\tan(x) + 7x}$

c) $\lim\limits_{x\to \infty} \left(\dfrac{x}{3+x}\right)^{14x}$

11) Juan Martin del Potro wants to build a rectangluar tennis court inside of a semi-circular arena of radius 43 feet, but he doesn't want to leave too much room for the players to move from side to side. Therefore he wants the corners of one side of the court to touch the edges of the arena and the other side to be on the edge of the arena. See the picture below:



a) Find a formula for the area of the tennis court in terms of its length and width.

b) Determine a one-variable formula for the area of the court.

c) Determine the dimensions of the court with maximum area. A correct answer without work will get you one point.