

Name:

Math 115 Practice Final

1) Compute the derivatives of the following functions.

a) $f(x) = x^4 \cos(x)$

b) $g(x) = \frac{x+1}{7x+2}$

c) $h(x) = \sec(x^3 + 9x)$

2) Find the equation of the tangent line to the graph of

$$f(x) = \sqrt{5x^2 - 1}$$

at the point $(1, 2)$.

3) Consider the function $f(x) = x^4 + 4x^3 - 144x^2$.

a) Find all local maxima, local minima, and intervals of increase/decrease for f .

b) Determine the intervals of concavity and inflection points (if any exist) for f .

4) Evaluate the following integrals.

a) $\int (10x^4 - 2x) dx$

b) $\int x^2 \csc^2(x^3) dx$

c) $\int_{-1}^2 \frac{x^2 + 1}{\sqrt{2 + x}} dx$

5) Find the value of the limits, if they exist.

a) $\lim_{x \rightarrow 1} \left(\tan \left(\frac{\pi}{4} x \right) - 6x \right)$

b) $\lim_{x \rightarrow 5} \frac{|x - 5|}{x^2 - 25}$

c) $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^3}$

6) a) Define what it means for a function f to be continuous at a point $x = a$

b) Find all values of k (if any exist) that make the function

$$f(x) = \begin{cases} -x + 8 & x < 7 \\ x^2 - 6x - 6 & x > 7 \\ k & x = 7 \end{cases}$$

continuous at $x = 7$. Be sure to show your work and be sure that your work is in accord with the definition provided in part a)!

7) Find the area enclosed by the y -axis and the graphs of $y = \cos(x)$ and $y = \sin(x)$ for $x \geq 0$.

8) You are painting the side of your house on a straight ladder which is 8 ft tall when Sasquatch (who does not appreciate your environmentally unfriendly paint) appears and begins to pull the base of the ladder away from your house, with you still on it.

a) Draw a picture representing the above scenario, labeling your variables.

b) If Sasquatch pulls the base of the ladder away at a constant rate of 2 ft/s, how fast is the angle between the top of the ladder and the side of your house changing when the base of the ladder is 4 feet away from the side of your house?

9) Consider the region bounded above by $y = \sqrt{x-5}$ and below by $y = (x-5)^{1/3}$ from $x = 5$ to $x = 6$.

a) Draw a picture of the region.

b) Find an integral representing the volume obtained by revolving the region about the x -axis

c) Compute the volume of the solid obtained in part b).

11) A rectangle is inscribed in a right triangle with legs of length 10 cm and 4 cm with two sides of the rectangle lying along the legs of the triangle.

a) Draw a picture that reflects this scenario.

b) Establish an equation in one variable for the area of the rectangle (*Hint: similarity*).

c) Find the dimensions of the rectangle with the largest possible area satisfying the given conditions.