

Math 115 Final

April 23rd, 2014

1) Compute the derivatives of the following functions.

a) (6 points) $f(x) = 4\sqrt{x} \tan(x)$

b) (6 points) $g(x) = \frac{11 - x^2}{9 + 3x}$

c) (8 points) $h(x) = (\cos(4x))^{1/3}$

2) (11 points) Find the equation of the tangent line to the graph of

$$\sin(x + y) - 2xy = 0$$

at the point $(0, \pi)$.

3) Consider the function $f(x) = x^7 - 7x$.

a) (10 points) Find the absolute maximum and minimum of f on the interval $[-1/2, 3/2]$.

b) (8 points) Determine the intervals of concavity and inflection points (if any exist) for f .

4) Evaluate the following integrals.

a) (5 points) $\int_{-1}^8 (20x^4 - x^{7/3} + 1) dx$

b) (7 points) $\int \frac{2x + 1}{\sqrt[4]{x^2 + x + 17}} dx$

c) (9 points) $\int_{\pi/4}^{\pi/3} \sin^3(x) dx$

5) Find the value of the limits, if they exist.

a) (4 points) $\lim_{x \rightarrow 4} \frac{|x - 2|}{2x - 4}$

b) (7 points) $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + x - 9}}{17x + 2}$

c) (10 points) $\lim_{x \rightarrow 0} \frac{\cos^2(x) - 1}{x^3 + 3x^2}$

6) Let

$$f(x) = \begin{cases} k^2x - 3k - 10 & x > 1 \\ 2kx + 4 & x < 1 \\ 18 & x = 1. \end{cases}$$

a) (8 points) Find all values of k (if any exist) such that f has a limit at $x = 1$.

b) (8 points) Find all values of k (if any exist) that make the function f continuous at $x = 1$.

7) Let $f(x) = x \cos(x) - x^2$.

a) (6 points) Show that f has a real zero (root) in the interval $(0, \pi/2]$.

b) (7 points) Compute a left sum using four equal subdivisions, accurate to four decimal places, that approximates $\int_0^{\pi/2} f(x) dx$.

8) Let $f(x) = \cos(x^2)$. Let \mathcal{R} denote the region between the graph of f and the x -axis from $x = 0$ to $x = \pi/2$.

a) (4 points) Set up an integral representing the area of \mathcal{R} BUT DO NOT EVALUATE THE INTEGRAL.

c) (6 points) Set up an integral for the volume obtained by revolving \mathcal{R} about the y -axis BUT DO NOT EVALUATE THE INTEGRAL.

9) Sasquatch is pouring his favorite ecologically friendly beverage into an inverted right conical drinking cup at the rate of $3 \text{ in}^3/\text{s}$. The height of the cup is 12 inches and has a maximum radius of 4 inches at the top. The volume of a cone is $\frac{\pi r^2 h}{3}$.

a) (5 points) Draw a picture that reflects this scenario, labeling your variables.

b) (8 points) Find an equation that relates the volume of the amount of liquid in the cup at a given time to the height of the liquid (no r 's should appear in this equation).

c) (12 points) Determine how fast the height of the liquid is changing 2 seconds after Sasquatch begins to pour.

10) Kitty Kowalski is helping Lex Luthor escape from prison, but she is confounded by a 20 foot tall wall that parallels the prison 10 feet away on every side. Her getaway plan consists of propping a ladder against the wall so that the top rests on the side of the prison and the bottom rests on the ground. However, Kitty doesn't know what size of ladder to get.

a) (5 points) Draw a picture representing the above scenario, labeling your variables.

b) (8 points) Find an equation in one variable for the length of the ladder (*Hint*: similarity).

c) (12 points) Determine the shortest possible ladder that will reach from the ground over the fence and to the side of the prison (*Hint*: it's good enough to minimize the square of the length).