Winter 2010 Exam 3

1) a) $\int_1^3 4x^3+1\,dx$

$= (x^4+x)\Big|_1^3$

$= 84-2 = 82$

b) $\int \tan(x)\sec^2(x)\,dx$

$= \dfrac{\tan^2(x)}{2} + C$

(technically, there is a different c for every interval of length π where tangent has no discontinuities)

c) let $u = 2+x$, $x = u-2$

$du = dx$

$u(-1) = 1$

$u(2) = 4$

$$\int_{-1}^{2} \frac{x^2+1}{\sqrt{2+x}}\,dx = \int_{1}^{4} \frac{(u-2)^2+1}{\sqrt{u}}\,du$$

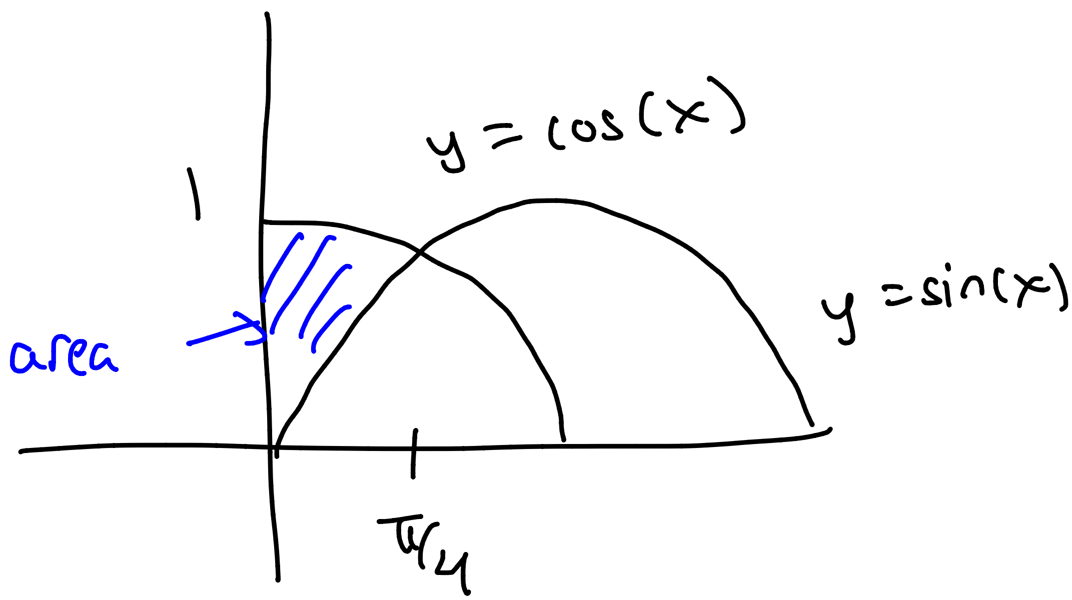
$$= \int_{1}^{4} \frac{u^2-4u+5}{\sqrt{u}}\,du$$

$$= \int_{1}^{4} u^{3/2} - 4u^{1/2} + 5u^{-1/2}\,du$$

$$= \left(\frac{2u^{5/2}}{5} - \frac{8u^{3/2}}{3} + 10u^{1/2}\right)\Big|_{1}^{4}$$

$$= \frac{56}{15}$$

2)



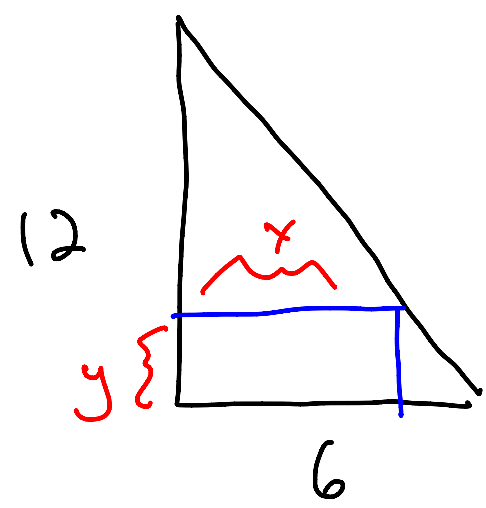
intersection at $x = \pi/4$

$$A = \int_0^{\pi/4} (\cos(x) - \sin(x))\,dx$$

$$= \left(\sin(x) + \cos(x) \right) \Big|_0^{\pi/4}$$

$$= \sqrt{2} - 1$$

3)



By similarity,

$$\frac{12}{12-y} = \frac{6}{x}$$

$$x = \frac{1}{2}(12-y)$$

$$A = xy = \frac{1}{2}(12y - y^2)$$

$$A'(y) = \frac{1}{2}(12 - 2y)$$

$$0 = A'(y) = \frac{1}{2}(12-2y) = 6 - y$$

$$y = 6$$

If $y=6$, $x=\frac{1}{2}(12-6)$

$=3$

Note $A''(y)=-1<0$,

so we have a local max

by the second derivative test.

4) a) $f(0) = \cos(0) = 1 > 0$

$f(-1) = \cos(-\pi) - 18 = -19 < 0$

So by the intermediate value theorem, there is a zero in the interval $[-1, 0]$.

b) Suppose f had two zeros x_1 and x_2 with $x_1 < x_2$. Then by the Mean Value Theorem, there is a point c in $[x_1, x_2]$ with

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = 0$$

But

$$f'(x) = -\pi \sin(\pi x) + 18x^2 + 12$$

$$\geq 12 - \pi > 0$$

so f' can't equal zero, and

so we can't have two zeros

x_1 and x_2.

c) $$X_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= -\frac{1}{2} - \frac{f(-1/2)}{f'(-1/2)}$$

$$= -\frac{1}{2} + \frac{27}{4\left(\frac{33}{2}+\pi\right)}$$

$$X_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= -\frac{1}{2} + \frac{27}{4(33/2+\pi)} - \frac{f\left(-\frac{1}{2} + \frac{27}{4\left(\frac{33}{2}+\pi\right)}\right)}{f'\left(-\frac{1}{2} + \frac{27}{4\left(\frac{33}{2}+\pi\right)}\right)}$$

$$\approx -.0832728$$