

Calculus Without Tears

Series Overview:

Volume 1 – Constant Velocity Motion

Volume 2 – Newton’s Apple

Volume 3 – Nature’s Favorite Functions

Table of Contents – Volume 1

_1. The Mathematics of Motion – Calculus is the mathematics of change. Tables, mathematical expressions, and functions can be used to represent change.

_2. Functions and Graphs – Graphs are ‘pictures’ of functions. We learn how to draw the graphs of functions representing constant velocity motion.

_3. Velocity – When a function represents motion, then the ‘rate of change’ of the function is the velocity of that motion. Calculus is all about calculating velocities, and it’s easy for constant velocity motion. The rate of change of a function is called its derivative; differential calculus is the study of derivatives.

_4. The Area Under a Curve – Differential calculus is about calculating velocities, integral calculus is about the inverse problem, that is, given a function representing velocity, determine position. Again, easy for constant velocity motion. And, there is a big bonus; according to the Fundamental Theorem of Calculus the area under a velocity curve for an interval equals distance traveled in that interval. We’ll prove it.

_5. Differential Equations – Modern math and science started with a single differential equation, $F = M \cdot A$. We will solve a special case, when F (force) is zero. In Volume 2 we will solve the case when F is the force of gravity (this is the case Newton solved to explain the workings of the world).

Berkeley Science Books
529 Bonnie Dr.
El Cerrito CA 94530
510-524-8094
www.berkeleyscience.com

ISBN 0-9764138-0-9
Copyright © 2004 by William Flannery

Introduction:

These lessons are a revolutionary approach to learning calculus. There is no algebra, no trigonometry, and no geometry (beyond the formula for the area of a rectangle). Why? - because they are not needed. Calculus is the mathematics of change, and change is represented by functions. The basic operations in calculus are differentiating and integrating functions, and solving differential equations. If the functions and equations are easy, there is no need for any algebra or trigonometry at all. The approach in these lessons is to learn calculus using easy functions; once the fundamentals are understood using easy functions, only then are more complex functions studied.

Modern math and science started with a single differential equation. In the 17th century Isaac Newton discovered gravity, and wrote the differential equation ($\text{Force} = \text{Mass} \cdot \text{Acceleration}$) that explained the motion of the moon and the planets. This equation launched the scientific and technological revolution that has transformed our world. It is the basis for physics, and is used every time motion is analyzed, from the calculation of the trajectory for the Apollo spacecraft, to the design of the rotor in your electric toothbrush.

The motivation for all mathematics beyond arithmetic is physics, and physics begins with differential equations (see previous paragraph). Yet in secondary school we are teaching complex algebra, geometry, and trigonometry before teaching the physics necessary to motivate their study. The current math curriculum is upside down. Teaching calculus early it will make it possible to study problems from physics and electronics (circuit analysis also starts with differential equations) that will motivate the entire math and science curriculum.

These lessons were written to teach calculus to a student in the 4th grade. The formal prerequisite is decimal arithmetic, that is, adding, subtracting, multiplying, and dividing easy decimal numbers. Some familiarity with rate problems is also desirable (e.g., if a car travels at 15 miles per hour, how long does it take the car to travel 45 miles?). Surprisingly, the fundamentals of calculus are easy and intuitive. Here is a shocker: differentiation is a generalization of the formula $\text{velocity} = \text{distance} / \text{time}$, and integration is a generalization of the formula $\text{distance} = \text{velocity} \cdot \text{time}$! The presentation is rigorous in essence but not weighted down by technical details. The goal is for the student to understand calculus and differential equations the way someone who works with them every day understands them, with a good intuitive grasp of the fundamental concepts.

William Flannery
Berkeley, CA, 2004