Things You Should Know For the Second Exam

Exam covers 1.8, 2.1-2.8, 3.1, 3.3-3.5

1) The Intermediate Value Theorem: the statement and how to the theorem to show a function has a zero.

2) Differentiation

- The definition of the derivative, in both its forms.
- The geometric interpretation of the derivative as the slope of the tangent line
- The equation of the tangent line: $f'(a)(x-a) = y - f(a)$
- The interpretation of the derivative as velocity, the second derivative as acceleration, and the third derivative as "jerk."
- The product rule $(f(x)g(x))' = f(x)g'(x) + g(x)f'(x)$ and how to use it
- The chain rule $(f \circ g)'(x) = f'(g(x))g'(x)$ and how to use it
- The quotient rule $\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$ and how to use it (if you want to)

- The trig derivatives

  $\frac{d}{dx}(\sin(x)) = \cos(x) \quad \frac{d}{dx}(\tan(x)) = \sec^2(x) \quad \frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$

  $\frac{d}{dx}(\cos(x)) = -\sin(x) \quad \frac{d}{dx}(\cot(x)) = -\csc^2(x) \quad \frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$

  In particular, remember that there is a negative sign accompanying the "co" derivatives.

- Implicit differentiation
  - how to do it
  - how to find the tangent line to an equation
  - related rates: every one is implicit differentiation, usually with respect to time!
    * draw a picture;
    * label the variables and identify what you're looking for and what you're given;
    * find an equation that relates all the quantities, implicitly differentiate it, THEN plug in the numbers;
    * solve!
    * some things to keep in mind: the pythagorean theorem, similar triangles, the definitions of trigonometric functions, the volumes of a cone and sphere.

- Maximum/minimum values of a function.
  - the difference between an absolute maximum (or absolute minimum) and a local maximum (or local minimum) for a function
  - a continuous function on a closed interval $[a, b]$ attains its absolute maximum and minimum values
    * to find the maximum and minimum, calculate all points in $[a, b]$ where $f'(x) = 0$ or $f'(x)$ does not exist (i.e. the critical numbers);

    * plug all these points and the endpoints $x = a$ and $x = b$ back into the ORIGINAL FUNCTION $f(x)$; the biggest value of f is the maximum and the smallest is the minimum,
  - Fermat's theorem: if f has a local maximum or minimum at $x = a$, then either $f'(a) = 0$ or $f'(a)$ does not exist.

- Intervals of increase/decrease for a function f
  - find all points where $f'(x) = 0$ or $f'(x)$ does not exist;
  - plot these points on a number line- this determines a collection of open intervals;
  - take points in the open intervals and plug the into the DERIVATIVE $f'(x)$. A positive value for $f'(x)$ means f is increasing on that interval, a negative value means f is decreasing;
  - use the intervals of increase/decrease to determine whether a function has a local maximum, minimum, or neither (the first derivative test)

- The second derivative test: if $f'(c) = 0$, then
  - $f''(c) > 0$ means f has a local minimum at $x = c$;
  - $f''(c) < 0$ means f has a local maximum at $x = c$;
  - $f''(c) = 0$ means the test fails and you know nothing.

- Intervals of concavity for a function f
  - find all points where $f''(x) = 0$ or $f''(x)$ does not exist;
  - plot these points on a number line- this determines a collection of open intervals;
  - take points in the open intervals and plug the into the SECOND DERIVATIVE $f''(x)$. A positive value for $f''(x)$ means f is concave up on that interval, a negative value means f is concave down;
  - use the intervals of concavity to determine whether a function has an inflection point

3) Limits: the identity $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ and how to use it.

4) Any and all algebra and trigonometry that you have ever learned (minus some of the weirder trig identities).