Things You Should Know For Final

Exam covers 1.4-1.6, 1.8, 2.1-2.8, 3.1-3.5, 3.7-3.9, 4.1-4.5, 5.1, 5.2

1) Limits

- The idea of a limit, limit from the left, and limit from the right; how to tell if any of these exist by looking at a graph. You do NOT need to know the precise definitions involving epsilons.
- The limit laws: know them! In particular, be able to tell when something is NOT a limit law!
- How to algebraically manipulate expressions of the form 0/0 in order to find a limit (using factorization, common denominators, etc.) OR
- l'Hopitals's rule: only for 0/0 or $\pm\infty/\pm\infty$ quotients; in that case and provided the necessary hypotheses are met,

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)};$$

  it will also work for $0 \times \pm\infty$ provided you rewrite the product as a quotient.
- The identity $\lim_{x\to 0}\frac{\sin(x)}{x} = 1$ and how to use it.
- Vertical and Horizontal Asymptotes: how to find them and what they mean for a graph. For vertical, know that these exist whenever you see a limit of the form $x/0$ where $x \neq 0$.
- Limits to Infinity: be aware of the conjugation trick when you have $\infty - \infty$. You may freely use the result that $\lim_{x\to\pm\infty}\frac{1}{x} = 0$. You do NOT need to know the precise definitions involving epsilons.

- Infinite Limits: be able to distinguish whether a limit (from the right, from the left, or in general) is infinity, negative infinity, or does not exist. You do NOT need to know the precise definitions involving epsilons.
- The Squeeze Theorem: know how to state it and in what situations it applies (usually sines and cosines are involved)

2) Continuity

- Know the definition.
- Determine where an (algebraically given) function is continuous- especially a piecewise-defined one!
- Be able to check continuity from a graph.
- You may use the fact that a polynomial is continuous on all real numbers and a rational function is continuous wherever it is defined.

3) The Intermediate Value Theorem: the statement and how to use the theorem to show a function has a zero.

4) Differentiation

- The definition of the derivative, in both its forms.
- The geometric interpretation of the derivative as the slope of the tangent line
- The equation of the tangent line: $f'(a)(x-a) = y - f(a)$
- The interpretation of the derivative as velocity, the second derivative as acceleration, and the third derivative as "jerk."
- The product rule $(f(x)g(x))' = f(x)g'(x) + g(x)f'(x)$ and how to use it
- The chain rule $(f \circ g)'(x) = f'(g(x))g'(x)$ and how to use it
- The quotient rule $\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$ and how to use it (if you want to)

- The trig derivatives

$$\frac{d}{dx}(\sin(x)) = \cos(x) \quad \frac{d}{dx}(\tan(x)) = \sec^2(x) \quad \frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x) \quad \frac{d}{dx}(\cot(x)) = -\csc^2(x) \quad \frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$$

  In particular, remember that there is a negative sign accompanying the "co" derivatives.

- Implicit differentiation
  - how to do it
  - how to find the tangent line to an equation
  - related rates: every one is implicit differentiation, usually with respect to time!
    * draw a picture;
    * label the variables and identify what you're looking for and what you're given;
    * find an equation that relates all the quantities, implicitly differentiate it, THEN plug in the numbers;
    * solve!
    * some things to keep in mind: the pythagorean theorem, similar triangles, the definitions of trigonometric functions, the volumes of a cone and sphere.

- Maximum/minimum values of a function.
  - the difference between an absolute maximum (or absolute minimum) and a local maximum (or local minimum) for a function
  - a continuous function on a closed interval $[a, b]$ attains its absolute maximum and minimum values
    * to find the maximum and minimum, calculate all points in $[a, b]$ where $f'(x) = 0$ or $f'(x)$ does not exist (i.e. the critical numbers);
    * plug all these points and the endpoints $x = a$ and $x = b$ back into the ORIGINAL FUNCTION $f(x)$; the biggest value of f is the maximum and the smallest is the minimum,
  - Fermat's theorem: if f has a local maximum or minimum at $x = a$, then either $f'(a) = 0$ or $f'(a)$ does not exist.
- Intervals of increase/decrease for a function f
  - find all points where $f'(x) = 0$ or $f'(x)$ does not exist;
  - plot these points on a number line- this determines a collection of open intervals;
  - take points in the open intervals and plug the into the DERIVATIVE $f'(x)$. A positive value for $f'(x)$ means f is increasing on that interval, a negative value means f is decreasing;
  - use the intervals of increase/decrease to determine whether a function has a local maximum, minimum, or neither (the first derivative test)
- The second derivative test: if $f'(c) = 0$, then
  - $f''(c) > 0$ means f has a local minimum at $x = c$;
  - $f''(c) < 0$ means f has a local maximum at $x = c$;
  - $f''(c) = 0$ means the test fails and you know nothing.

- Intervals of concavity for a function f
  - find all points where $f''(x) = 0$ or $f''(x)$ does not exist;
  - plot these points on a number line- this determines a collection of open intervals;
  - take points in the open intervals and plug the into the SECOND DERIVATIVE $f''(x)$. A positive value for $f''(x)$ means f is concave up on that interval, a negative value means f is concave down;
  - use the intervals of concavity to determine whether a function has an inflection point

5) The Mean Value Theorem: the statement and how to use the theorem to show a function has exactly one zero.

6) Antiderivatives (same as indefinite integrals)

- How to compute them.
- Any two antiderivatives differ by a constant.

7) Newton's Method: how to use it, when it fails.

8) Optimization problems

- Draw a picture and label it
- Determine an equation in one variable for the quantity you want to optimize
- Take the derivative of the equation, set it equal to zero
- If necessary, use either the first or second derivative tests to check that your answer actually satisfies the required properties

9) Integration

- The definition of a definite integral

$$\int_a^b f(x)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{b-a}{n} f\left(a + \frac{i(b-a)}{n}\right)$$

  provided the limit exists!

- How to use an integral to find the area between two curves, especially if one of those curves is the x-axis
  - find all points where the curves intersect by setting them equal to eachother
  - determine which function is bigger between two consecutive points where they are equal and integrate the bigger one minus the smaller one over those points
  - add all the answers to get the area

- Properties of integrals. The first two hold for indefinite integrals as well.

  1. $\int_a^b (f(x) \pm g(x))\ dx = \int_a^b f(x)\ dx \pm \int_a^b g(x)\ dx$
  2. $\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$ if "c" is a constant
  3. $\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$ if $a \leq c \leq b$
  4. $\int_a^a f(x)\ dx = 0$
  5. if $f \leq g$ on $[a, b]$, then $\int_a^b f(x)\ dx \leq \int_a^b g(x)\ dx$.

- The Fundamental Theorem of Calculus: know the statement (both parts) and how to use it.

- Substitution: how to use it, for both definite and indefinite integrals

- Volume problems:
  - (Disk method) The formula for the volume obtained by revolving the region between two curves $f(x)$ and $g(x)$ from $x = a$ to $x = b$ about the x-axis when $f \geq g$ is

    $$V = \int_a^b \pi(f(x))^2 \, dx - \int_a^b \pi(g(x))^2 \, dx.$$

    Know this formula and know how to use it, ESPECIALLY if g is the x-axis.
  - (Shell method) The formula for the volume obtained by revolving the region between two curves $f(x)$ and $g(x)$ from $x = a$ to $x = b$ about the y-axis when $f \geq g$ is

    $$V = \int_a^b 2\pi x f(x) \, dx - \int_a^b 2\pi x (g(x))^2 \, dx.$$

    Know this formula and know how to use it, ESPECIALLY if g is the x-axis.

10) Any and all algebra and trigonometry that you have ever learned (minus some of the weirder trig identities).