Math 116 Exam 1

October 2, 2014

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.

True/False. If the sentence is false, correct the error. No justification is necessary.

a) (2 points) $\lim\limits_{x \to 0^+} \ln(x) = \infty$.

b) (2 points) Using partial fractions, there are numbers A and B with

$$\frac{3x^6 - x + 1}{(x - 11)(x + 2)} = \frac{A}{x - 11} + \frac{B}{x + 2}.$$

c) (2 points) For all positive real numbers x and y, $e^{x/y} = e^x - e^y$.

d) (2 points) For all real numbers x, $\sin^2(6x) = \dfrac{1 + \cos(12x)}{2}$.

e) (2 points) The domain of arctangent is $[0, \pi]$.

2) Find the first derivative for the following functions.

a) (6 points) $f(x) = e^{x^3 + x}$

b) (8 points) $h(x) = \arctan(\ln(x)), \ x > 0$

c) (10 points) $g(x) = x^{x^2}, \ x > 0$

3) Evaluate the following indefinite integrals.

a) (10 points) $\displaystyle\int \frac{3x+2}{x^2 - 8x + 12}\, dx$

b) (10 points) $\displaystyle\int 5x^2 \cdot e^{-7x}\, dx$

4) Compute the following limits.

a) (6 points) $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{\sin(3x)}$

b) (8 points) $\lim\limits_{x \to \pi/2^-} (\tan(x) \ln(\pi - 2x + 1))$

c) (10 points) $\lim\limits_{x \to \infty} \left(\dfrac{\sqrt{x}}{6 + \sqrt{x}} \right)^{\sqrt{x}}$

5) Evaluate the following definite integrals.

a) (10 points) $\displaystyle\int_1^{\sqrt{e}} \frac{1}{x\sqrt{1-(\ln(x))^2}} \, dx$

b) (12 points) $\displaystyle\int_0^{\pi/4} \tan(x)\cos(2x) \, dx$

BONUS: (10 points) DO NOT ATTEMPT THIS PROBLEM UNTIL YOU ARE DONE WITH THE REST OF THE EXAM!

Find $\displaystyle\int e^x \ln(x) \; dx$.