Name:

Math 116 Exam 1

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.

1) A mixture of water and sugar flows into a tank containing 800L of pure Michigan water. Let $x(t)$ denote the amount of sugar in the tank at time t. Suppose the mixture flows into the tank at 6L/min and flows out at the same rate.

a) (2 points) Determine the initial condition on x.

b) (10 points) Find an equation for $\dfrac{dx}{dt}$ in terms of $x(t)$ if the mixture flowing in contains .5kg/L of sugar.

c) (6 points) How does the equation in part b) change if, after one hour, the concentration of the mixture changes to 1.5kg/L of sugar?

2) Your can of Mountain Dew Code Red® is a lovely 80°F, but you would like to drink it at a less tepid temperature, so you put it in the freezer. The temperature inside your freezer is 0°F. You take the can out after 8 minutes and the temperature registers 60°F, which is still too warm, so you immediately put it back in the freezer.

a) (15 points) Find an explicit formula for the temperature $f(t)$ of the can of The Dew.

b) (5 points) At what time will you need to pull the can of The Dew out if you want it to be 38°F when you begin drinking it?

3) a) (3 points) If f is continuous on $[0, \infty)$, define $\displaystyle\int_0^\infty f(t)\ dt$.

b) (7 points) State L'Hôpital's Rule.

c) (20 points) Compute the Laplace Transform of $f(t) = 2t - 3$. Recall that the Laplace Transform of a function f is defined as

$$\mathcal{L}\{f\}(w) = \int_0^\infty f(t)e^{-wt}\ dt.$$

4) a) (11 points) Find the partial fraction decomposition of $\dfrac{2x+1}{x^3+x}$.

b) (10 points) Compute $\lim\limits_{x\to\infty}\left(1+\dfrac{\ln(2)}{x}\right)^x$

c) (11 points) Determine the value of $\displaystyle\int_{\pi/6}^{\pi/4}\frac{\sec(t)}{\cot(t)+\sec(t)\csc(t)}\,dt$