Name:

Math 116 Exam 1

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.

1) A tank contains 300L of water with 1.5 kg of dissolved glucose initially present. A mixture containing water with .05 kg/L of glucose flows into the tank at a rate of 20L/min and flows out at the same rate. If you are pedantic, the mixture is kept uniform by stirring. Let $x(t)$ denote the amount of sugar in the tank at time t.

a) (10 points) Find an equation for $\dfrac{dx}{dt}$ in terms of $x(t)$, plugging in all relevant numbers.

b) (12 points) Solve the equation you found in part a) for $x(t)$.

c) (4 points) Find the amount of glucose in the tank after 30 minutes.

2) Bigfoot is boiling some Campbell's® soup. He would like the soup to be about 160°F before he begins eating. Of course, he is cooking outdoors, where the temperature is holding steady at 60°F. He takes the soup off of the flame and waits for 5 minutes, after which he tastes it and it still burns his tongue. Bigfoot's careful senses discern the soup is 180° when it crisps his mouth. Let $f(t)$ denote the temperature of the soup at time t.

a) (6 points) State Newton's Law of Cooling.

b) (2 points) Fill in any constants you can in the equation you found in a).

c) (6 points) Do you have enough information to tell Bigfoot when he should eat his soup? Why or why not?

3) a) (3 points) If f is continuous on $[0, \infty)$, define $\displaystyle\int_0^\infty f(t)\ dt$.

b) (10 points) Compute the Laplace Transform of $f(t) = e^{-6t}$. Recall that the Laplace Transform of a function f is defined as

$$\mathcal{L}\{f\}(s) = \int_0^\infty f(t)e^{-st}\ dt.$$

c) (5 points) For which values of s does the Laplace Transform of e^{-6t} exist? Why?

4) Evaluate the following limits:

a) (8 points) $\displaystyle\lim_{x \to 1} \frac{\tan(x^2 - 1)}{x^3 - 1}$

b) (12 points) $\displaystyle\lim_{x \to \infty} \left(1 + 5e^{-x}\right)^{2e^x}$

5) Compute the following integrals:

a) (12 points) $\displaystyle\int t^2 \sin(7t)\ dt$

b) (10 points) $\displaystyle\int_1^4 \frac{1}{\sqrt{t}+1}\ dt$