

Name:

Math 116 Exam 1

February 7, 2018

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) A tank contains 750L of water with 5 kg of dissolved fructose initially present. A mixture containing water with .02 kg/L of fructose flows into the tank at a rate of 10L/min and flows out at the same rate. If you are pedantic, the mixture is kept uniform by stirring. Let $x(t)$ denote the amount of sugar in the tank at time t , in kilograms.

a) (6 points) Suppose someone tells you that $x(15) = 4$ kg. Is this possible? Why or why not?

b) (4 points) Without solving for $x(t)$, what is the value of $\lim_{t \rightarrow \infty} x(t)$? Justify your answer with some reasoning.

2) A tank contains 750L of water with 5 kg of dissolved fructose initially present. A mixture containing water with .02 kg/L of fructose flows into the tank at a rate of 10L/min and flows out at the same rate. If you are pedantic, the mixture is kept uniform by stirring. Let $x(t)$ denote the amount of sugar in the tank at time t , in kilograms.

a) (10 points) Find an equation for $\frac{dx}{dt}$ in terms of $x(t)$, plugging in all relevant numbers.

b) (12 points) Solve the equation you found in part a) for $x(t)$.

c) (4 points) Find the amount of fructose in the tank after 15 minutes.

3) Your Ford Flex has spent the night in your attached garage and registers an exterior temperature of 24°F when you turn the ignition in the morning. As you drive, you note the exterior temperature dropping, and after one minute, it registers 13°F . After 3 minutes, it registers 9°F . Let $f(t)$ denote the temperature of the car's exterior at time t .

a) (6 points) State Newton's Law of Cooling.

b) (4 points) Do you have enough information to find a formula for the exterior temperature? Why or why not?

c) (2 points) Suppose you also know the exterior temperature of the car stabilizes after 3 minutes. Now do you have enough information? DO NOT ACTUALLY SOLVE!

4) Compute the answers for the following problems:

a) (8 points) $\lim_{x \rightarrow \infty} \frac{e^{2/x} - 1}{1/x}$

b) (12 points) $f'(\pi/4)$ if $f(x) = \cos(x)^{\sin(x)}$

c) (12 points) $\int x^3 \cos(2x) dx$.

5) a) (3 points) If f is continuous on $[0, \infty)$, define $\int_0^\infty f(t) dt$.

b) (12 points) Compute the Laplace Transform of $f(t) = 3^t$. Recall that the Laplace Transform of a function f is defined as

$$\mathcal{L}\{f\}(s) = \int_0^\infty f(t)e^{-st} dt.$$

c) (5 points) For which values of s does the Laplace Transform of 3^t exist? Why?