Exam 1 Winter 18

a) No. The amount per liter coming in is $.02 \frac{kg}{L} = \frac{1}{50}$ which is greater than

$\frac{5}{750} = \frac{1}{150}$, so the amount of fructose will only increase

b) It is $.02 \cdot 750$
$= 15$ kg

Since the concentration per liter will approach the concentration of the liquid flowing in.

2) a) $\frac{dx}{dt} = (\text{rate in}) - (\text{rate out})$

$= .02 \frac{kg}{L} \cdot 10 \frac{L}{min} - 10 \frac{L}{min} \frac{x(t)}{750} \frac{kg}{L}$

$= \left(.2 - \frac{x(t)}{75} \right) kg/min$

$= \frac{15 - x(t)}{75}$

b) $\frac{1}{15 - x(t)} \frac{dx}{dt} = \frac{1}{75}$

$\int \frac{1}{15 - x(t)} \frac{dx}{dt} dt = \int \frac{1}{75} dt$

$= \frac{t}{75} + C$

$$\int \frac{1}{15-x(t)} \frac{dx}{dt}\, dt$$

$$u = 15 - x(t)$$

$$du = -\frac{dp}{dt}\, dt$$

We get $-\int \frac{1}{u}\, du$

$$= -\ln(u)$$

$$= -\ln(15 - x(t))$$

$$= \frac{t}{15} + C$$

Since $x(0) = 5$,

$$-\ln(15-5) = C$$

$$C = -\ln(10)$$

$$-\ln(15-x(t)) = \frac{t}{75} - \ln(10)$$

$$\ln(15-x(t)) = -\frac{t}{75} + \ln(10)$$

$$\underbrace{e^{\ln(15-x(t))}} = \underbrace{e^{-\frac{t}{75}+\ln(10)}}$$

$$= 15 - x(t) = e^{-\frac{t}{75}} e^{\ln(10)}$$

$$= 10e^{-\frac{t}{75}}$$

$$x(t) = 15 - 10e^{-\frac{t}{75}}$$

c) $X(15) = 15 - 10e^{-\frac{15}{75}}$

$$= 15 - 10e^{-\frac{1}{5}}$$

$$\approx 6.813 \text{ kg}$$

3)

a) $\frac{df}{dt} = k(f(t) - T)$

k = constant of proportionality

T = ambient temp

b) Yes. We don't have the external temperature, but 3 temperature values at 3 different times will solve for k, T, and the constant of integration C.

c) Yes. Now you can take

$T = 9^\circ F$

4) a) $\lim_{x\to\infty} e^{\frac{2}{x}}-1 = 1-1=0$

$\lim_{x\to\infty} \frac{1}{x} = 0$

$$\lim_{x\to\infty} \frac{e^{2/x}-1}{\frac{1}{x}}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{-\frac{2}{x^2}e^{2/x}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} 2e^{2/x} = 2$$

b) $$f(x) = e^{\ln\left((\cos(x))^{\sin(x)}\right)}$$

$$= e^{\sin(x)\ln(\cos(x))}$$

$$f'(x) = e^{\sin(x)\ln(\cos(x))} \cdot \frac{d}{dx}\left(\sin(x)\ln(\cos(x))\right)$$

$$= e^{\sin(x)\ln(\cos(x))}\left(\cos(x)\ln(\cos(x)) + \sin(x)\cdot\frac{1}{\cos(x)}\cdot -\sin(x)\right)$$

$$= \cos(x)^{\sin(x)}\left(\cos(x)\ln(\cos(x)) - \frac{\sin^2(x)}{\cos(x)}\right)$$

$$f'(\pi/4) = \left(\frac{\sqrt{2}}{2}\right)^{\sqrt{2}/2}\left(\frac{\sqrt{2}}{2}\ln\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\right)$$

4) c) $\int x^3 \cos(2x)\,dx$

integrate by parts, tabular

	u		dv
	x^3	$+$	
			$\sin(2x)/2$
	$3x^2$	$-$	
			$-\cos(2x)/4$
	$6x$	$+$	
			$-\sin(2x)/8$
	6	$-$	
	0		$\cos(2x)/16$

$$= \frac{x^3 \sin(2x)}{2} + \frac{3x^2 \cos(2x)}{4} - \frac{6x \sin(2x)}{8}$$

$$- \frac{6 \cos(2x)}{16} + C$$

5) a) $\lim_{x \to \infty} \int_0^x f(t)\,dt$

b) $\mathcal{L}(3^t)(s)$

$= \int_0^\infty 3^t e^{-st}\,dt$

$= \lim_{x \to \infty} \int_0^x 3^t e^{-st}\,dt$

$= \lim_{x \to \infty} \int_0^x e^{\ln(3^t)} e^{-st}\,dt$

$= \lim_{x \to \infty} \int_0^x e^{t\ln(3)} e^{-st}\,dt$

$= \lim_{x \to \infty} \int_0^x e^{t(\ln(3)-s)}\,dt$

$$= \lim_{x\to\infty} \left(\frac{e^{t(\ln(3)-s))}}{\ln(3)-s} \Big|_0^x \right)$$

$$(s \neq \ln(3))$$

$$= \frac{1}{\ln(3)-s} \lim_{x\to\infty} \left(e^{x(\ln(3)-s)} - 1 \right)$$

$$= \frac{1}{s-\ln(3)} \quad \text{if } s > \ln(3)$$

c) $s \geq \ln(3)$ you need $s > \ln(3)$ for the limit of $e^{x(\ln(3)-s)}$ to equal zero, and $s \neq \ln(3)$ to avoid division by zero.