

Name:

# Math 116 Exam 1

January 30, 2020

## Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, DO NOT convert irrational numbers such as  $\sqrt{3}$  or  $\pi$  into decimal approximations; just leave them as they are.
4. If you have a question, raise your hand or come up and ask me.

1) (15 points) Starting with the initial condition  $y(0) = 1$  (i.e. start at  $(0,1)$ ), use three iterations of Euler's Method with  $\Delta x = 1$  to sketch a solution to

$$\frac{dy}{dx} = \cos(\pi y) + x.$$

2) A tank contains 1200L of water with 2 kg of dissolved sucrose initially present. A mixture containing water with .05 kg/L of sucrose flows into the tank at a rate of 8L/min and flows out at the same rate. If you are pedantic, the mixture is kept uniform by stirring. Let  $x(t)$  denote the amount of sugar in the tank at time  $t$ , in kilograms.

a) (5 points) Suppose someone tells you that  $x(10) = 4$  kg. Is this possible? Why or why not?

b) (16 points) Find an equation for  $\frac{dx}{dt}$  in terms of  $x(t)$ , plugging in all relevant numbers.

c) (2 points) To four decimal points, find the amount of sucrose in the tank after 10 minutes.

**3)** a) (16 points) Compute the Laplace Transform of  $f(t) = e^{2\ln(7^t)}$ . Recall that the Laplace Transform of a function  $f$  is defined as

$$\mathcal{L}\{f\}(w) = \int_0^{\infty} f(t)e^{-wt} dt.$$

b) (5 points) For which values of  $w$  does the Laplace Transform of  $e^{2\ln(7^t)}$  exist? Why?

4) (16 points) Compute  $\int_1^e \frac{\ln(x)}{\sqrt{x}} dx$