Name:

Math 116 Exam 1

January 30, 2023

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.
4. If you have a question, raise your hand or come up and ask me.

1) Brandon wants to drink his Celestial Seasonings® tea, but unfortunately, his tea reached a roiling boil and is 212° F when it comes out of the kitchen microwave. The temperature in the kitchen is 70° F. Let $f(t)$ denote the temperature of the tea, where t is measured in minutes.

a) Suppose someone tells you that $f(20) = 68°$ F. Without solving for $f(t)$, explain whether or not this is possible.

b) Brandon wants to drink his tea when it cools down to 85° F. Do you have enough information to solve for how much time he has to wait? If so, solve for how much time he has to wait, and if not, explain why not.

2) Croak, the number one soda for frogs, is created by channeling water containing .2kg of sugar per liter into a scummy pond initially filled with 750L of water that someone has tainted with 50kg of sugar. The mix flows in at 12 L/min, flows out at the same rate, and the mixture is kept uniform by stirring. Call $s(t)$ the amount of sugar (in kilograms) in the tank at time t (in minutes).

a) Suppose I tell you that $s(18) = 45$. Without solving for $s(t)$, explain whether this is possible or not.

b) Calculate $s(0)$.

c) Again without solving for $s(t)$, what is the value of $\lim_{t\to\infty} s(t)$? Justify your answer with some reasoning.

3) With the same data as problem 2),

a) Find an equation for $\frac{ds}{dt}$ in terms of $s(t)$, plugging in all relevant numbers.

b) Solve the equation you found in part a) for $s(t)$.

c) Find the amount of sugar in the tank after 10 minutes.

4) Find the area between the curves $y = \dfrac{\ln(x^4)}{x}$ and $y = \dfrac{\ln(x^3)}{x}$ from the point that they intersect to the line $x = e^3$.