Exam 1 w 23' 12:33

1) a) No, because that is colder than the ambient temperature

b) $\frac{df}{dt} = k(f(t) - 70)$

$\frac{1}{f(t)-70} \frac{df}{dt} = k$

$\int \frac{1}{f(t)-70} \frac{df}{dt} dt = \int k \, dt$

$= kt + C$

$u = f(t) - 70$

$du = \frac{df}{dt}$

$\int \frac{1}{u} du = kt + C$

$\ln|u| = kt + C$

$$\ln|f(t)-70| = kt + C$$

$$f(t)-70 = e^{kt+C}$$

$t=0, \quad f(0)=212$

$$212-70 = e^{C}$$

$$C = \ln(142)$$

Not enough information to find k, so you can't solve

2) a) $50/750 = 1/15 < .1 = 1/10$

so the amount of sugar should be increasing over time, not decreasing, so this is not possible

b) $s(0) = 50$

c) over time, the concentration will get closer to the concentration of the mixture flowing in, so the limit is $.1 \times 750 = 75$

3) a) $\frac{ds}{dt} = (\text{rate in}) - (\text{rate out})$

$$\frac{ds}{dt} = .2\,ug/L \cdot 12\,L/min - 12\,L/min \cdot \frac{S(t)}{750}$$

$$\frac{ds}{dt} = 2.4\ ug/min - \frac{2\,S(t)}{125}\ ug/min$$

b) $\frac{ds}{dt} = \frac{-2}{125}\left(S(t) - 150\right)$

$$\frac{1}{S(t)-150}\frac{ds}{dt} = -\frac{2}{125}$$

$$\int \frac{1}{S(t)-150}\frac{ds}{dt}\,dt = \int -\frac{2}{125}\,dt$$

$u = S(t) - 150$

$du = \frac{ds}{dt}$

$$= -\frac{2t}{125} + C$$

$$\int \frac{1}{u}\,du = -\frac{2t}{125} + C$$

$$\ln|u| = -\frac{2t}{125} + C$$

$$\ln|s(t) - 150| = -\frac{2t}{125} + C$$

$$150 - s(t) = e^{-\frac{2t}{125} + C}$$

$$s(0) = 50 \quad , \quad 50$$

$$C = \ln(100)$$

$$150 - s(t) = e^{-\frac{2t}{125} + \ln 100}$$

$$s(t) = 150 - e^{-\frac{2t}{125} + \ln(100)}$$

$$s(t) = 150 - 100e^{-\frac{2t}{125}}$$

c) $s(10) = 150 - 100\, e^{-\frac{20}{125}} \approx 64.79$

4)

$$\frac{\ln(x^4)}{x} = \frac{\ln(x^3)}{x}$$

$$\frac{\ln(x^4)}{x} - \frac{\ln(x^3)}{x} = 0$$

$$\frac{1}{x}\left(\ln(x^4) - \ln(x^3)\right) = 0$$

either $\frac{1}{x} = 0$ (never happens)

or $\ln(x^4) - \ln(x^3) = 0$

$$\ln(x^4) - \ln(x^3) = 4\ln(x) - 3\ln(x)$$

$$= \ln(x)$$

$$\ln(x) = 0$$

$$x = 1$$

$$\int_1^{e^3} \left| \frac{\ln(x^4)}{x} - \frac{\ln(x^3)}{x} \right| dx$$

$$= \int_1^{e^3} \left| \frac{4\ln(x)}{x} - \frac{3\ln(x)}{x} \right| dx$$

$$= \int_1^{e^3} \frac{\ln(x)}{x}\, dx$$

$$u = \ln(x) \qquad u(1) = 0$$

$$du = \frac{1}{x}\, dx \qquad u(e^3) = 3$$

$$= \int_0^3 u\, du = \frac{u^2}{2} \Big|_0^3 = \frac{9}{2}$$