

Name:

Math 116 Exam 2

November 17, 2016

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.

1) Consider the power series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{5^n(n^3+n+1)}$.

- a) (3 points) What is the center of the series?
- b) (14 points) Find the radius of convergence of the series.
- c) (2 points) Given that the radius of convergence is 5, list two numbers other than the center for which the series converges.

2) (17 points) Let $f(x) = \sum_{n=0}^{\infty} \frac{(-3)^n x^{n+1}}{n!}$. Show that f satisfies the differential equation

$$y'' + 6y' + 9y = 0.$$

3) a) (6 points) Define what it means for $\sum_{n=1}^{\infty} a_n$ to converge to a real number.

b) (13 points) Using the definition of convergence for a series, find the sum of $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})$ or show the series diverges.

4) Find the sum of the series or show the series diverges.

a) (9 points) $\sum_{n=1}^{\infty} \frac{(-1)^n 8^n}{3^{2n}}$.

b) (11 points) $\sum_{n=0}^{\infty} \frac{(-16)^{n+3}}{(2n+1)!}$

5) (13 points) Determine the limit of the sequence

$$\left(\left(1 + \frac{8}{n} \right)^n \right)_{n=1}^{\infty} .$$

6) (12 points) Let $f(x) = \frac{3x + 6}{x^2 + 4x + 2}$. Find a power series representation for f .