

Name:

Math 116 Exam 2

March 15, 2018

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) a) (6 points) Find a parameterization for the Cartesian curve $x^2 + y^2 = 49$. Your parameterization should capture the whole curve, not merely a portion of it.

b) (9 points) Find a Cartesian equation (i.e. one in x and y) for the parametric curve $x(t) = e^{t-1} + 2$, $y(t) = e^{t-2} + 1$. As in part a), your equation should capture the whole curve.

c) (2 points) What is the graph of the function from part b)?

d) (3 points) What kind of trig substitution would you make to evaluate the integral $\int_0^1 \sqrt{1+x^2} dx$?

2) (20 points) Determine an equation for the tangent line to the parametric curve given by $x(t) = (t + 1)^{3t}$, $y(t) = \arctan(e^t)$ at the point $t = 0$.

3) Consider the parametric curve given by $x(t) = \tan(t) - t$, $y(t) = \ln(\cos(t))$.

a) (10 points) Set up an equation for the arclength of the curve from $t = 0$ to $t = \pi/6$.

b) (10 points) Find the arclength of the portion of the curve described in part a).

4) Consider the parametric curve given by $x(t) = 7 \sin(t)$, $y(t) = 7 \cos(t)$.

a) (10 points) Set up an equation for the surface area obtained by revolving the curve from $t = 0$ to $t = 2\pi$ about the x -axis.

b) (5 points) Find the surface area of the surface obtained by revolving the curve from $t = 0$ to $t = 2\pi$ about the x -axis.

c) (5 points) Is the answer you got from part b) actually the surface area of the object you obtain by revolving the curve about the x -axis? Why or why not?

5) The *logistic equation* is sometimes used to model population growth. The equation is

$$\frac{df}{dt} = kf(t) \left(1 - \frac{f(t)}{K}\right)$$

where f is the population in a given region, K is the carrying capacity of the region in question, and k is a constant. For this problem, let $K = 1000$ and $k = .5$.

- a) (2 points) Write down the equation when $k = .5$ and $K = 1000$.
- b) (3 points) Can you solve this problem by integrating both sides directly? Why or why not?
- c) (15 points) Carry out a procedure to solve the problem, up to the point where you actually have to integrate.