1 a) $x = 7\cos\theta,\ y = 7\sin\theta$

b) $x = e^{t-1} + 2$

$y = e^{t-2} + 1$

$e^{t-1} = x - 2$

$y = e^{t-1} e^{-1} + 1$

$= (x-2)e^{-1} + 1$

$=$

c) a line (or more properly, a ray)

d) $x = \tan(\theta)$

2)

$$x(t) = e^{\ln(t+1)^{3t}}$$

$$= e^{3t\ln(t+1)}$$

$$x'(t) = e^{3t\ln(t+1)}\left(\frac{3t}{t+1} + 3\ln(t+1)\right)$$

$$= (t+1)^{3t}\left(\frac{3t}{t+1} + 3\ln(t+1)\right)$$

$$y'(t) = \frac{1}{1+e^{2t}} \cdot e^{t}$$

$x(0) = 1$ $\quad x'(0) = 0$

$y(0) = \pi/4$ $\quad y'(0) = 1/2$

$x'(0)=0$ gives a vertical line

$$x = 1$$

3) a) $x'(t) = \sec^2(t) - 1$

$$y'(t) = \frac{1}{\cos(t)} \cdot -\sin(t)$$

$$\int_0^{\pi/6} \sqrt{(\sec^2(t)-1)^2 + \left(\frac{-\sin(t)}{\cos(t)}\right)^2}\, dt$$

b) $$L = \int_0^{\pi/6} \sqrt{(\tan^2(t))^2 + \tan^2(t)}\, dt$$

$$= \int_0^{\pi/6} \sqrt{(\tan^2(t))\,(\tan^2(t)+1)}\, dt$$

$$= \int_0^{\pi/6} \sqrt{\tan^2(t) \quad \sec^2(t)} \quad dt$$

$$= \int_0^{\pi/6} \sec(t)\tan(t)\, dt$$

$$= \sec(t)\Big|_0^{\pi/6} = \frac{2}{\sqrt{3}} - 1$$

4) a) $y'(t) = -7\sin(t)$

$x'(t) = 7\cos(t)$

$$2\pi\left(\int_0^{2\pi} 7\cos t)\sqrt{7^2\sin^2(t)+7^2\cos^2(t)}\,dt\right)$$

b) $$14\pi\int_0^{2\pi} \cos(t)\sqrt{49(\underbrace{\sin^2(t)+\cos^2(t)}_{=1}}\,dt$$

$$= 14\pi\int_0^{2\pi} \cos(t)\cdot 7\,dt$$

$$= 98\pi\ \sin(t)\Big|_0^{2\pi} = 0$$

c) NO! Surface is a sphere, so

the surface area should be

$$4\pi r^2 = 4\pi \cdot 49$$

$$= 196\pi$$

5) a) $\frac{df}{dt} = .5f(t)\left(1 - \frac{f(t)}{1000}\right)$

b) No. Don't know how to integrate right-hand side

c) Divide by $f(t)\left(1 - \frac{f(t)}{1000}\right)$

$$\frac{1}{f(t)} \frac{1}{1 - \frac{f(t)}{1000}} \frac{df}{dt} = .5$$

$$\int \underbrace{\frac{1}{f(t)} \frac{1}{1 - \frac{f(t)}{1000}}} \frac{df}{dt}\, dt = \int .5\, dt$$

$u = f(t)$

$du = \frac{df}{dt}\, dt$

$$\int \frac{1}{v} \frac{1}{1-\frac{v}{1000}} \, dv = \int .5 \, dt$$

$$\frac{1}{v\left(1-\frac{v}{1000}\right)} = \frac{A}{v} + \frac{B}{1-\frac{v}{1000}}$$

$$1 = A\left(1-\frac{v}{1000}\right) + Bv$$

$$v = 1000 \qquad v = 0$$

$$B = \frac{1}{1000} \qquad A = 1$$