Exam 2 W 23'

1) a) No!

$$\frac{A}{6-2} + \frac{B}{(6-s)^2} + \frac{Cs+D}{s^2+8}$$

b) $6\sin(\theta)$, $6\cos(\theta)\,d\theta$

c) e^x, $\cos(x)$ (or reversed)

2)

$$\frac{s^3+1}{s(s^2-4)} = \frac{s^2+1}{s(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2} + \frac{C}{s}$$

$$s^2+1 = A(s+2)\cdot s + B(s-2)\cdot s + C(s-2)(s+2)$$

$s=0$	$s=2$	$s=-2$
$1=-4C$	$5=8A$	$5=8B$
$C=-1/4$	$A=5/8$	$B=5/8$

$$\frac{s^2+1}{s(s^2-4)} = \frac{5/8}{s-2} + \frac{5/8}{s+2} - \frac{1/4}{s}$$

3) $\mathcal{L}[f](7) = \int_0^\infty (t+2)^2 e^{-7t}\,dt$

$$= \lim_{x\to\infty} \int_0^x (t+2)^2 e^{-7t}\,dt$$

u		dv
$(t+2)^2$	$+$	e^{-7t}
$2(t+2)$	$-$	$e^{-7t}/-7$
2	$+$	$e^{-7t}/49$
0		$e^{-7t}/-343$

$$\int_0^x (t+2)^2 e^{-7t}\,dt =$$

$$-\frac{(t+2)^2 e^{-7t}}{7} - \frac{2(t+2)e^{-7t}}{49} - \frac{2e^{-7t}}{343}\Big|_0^x$$

$$-\frac{(t+2)^2 e^{-7t}}{7} - \frac{2(t+2)e^{-7t}}{49} - \frac{2e^{-7t}}{343} \Bigg|_0^x$$

$$= \frac{-(x+2)^2}{7e^{7x}} - \frac{2(x+2)}{e^{7x} \cdot 49} - \frac{2}{343e^{7x}} + \frac{2}{343}$$

$$+ \frac{4}{7} + \frac{4}{49}$$

$$\lim_{x \to \infty} \frac{-(x+2)^2}{7e^{7x}} \overset{\text{l'H}}{=} \lim_{x \to \infty} \frac{-2(x+2)}{49e^{7x}}$$

$$\overset{\text{l'H}}{=} \lim \frac{-2}{363e^{7x}} = 0$$

so

$$\mathcal{L}[(t+2)^2](7) = \frac{2}{343} + \frac{4}{7} + \frac{4}{49}$$

4)

$u = e^x$

$du = e^x \, dx$

$u(0) = 1$

$u(\ln(\sqrt{3})) = \sqrt{3}$

$$\int_1^{\sqrt{3}} \frac{du}{\sqrt{u^2+1}}$$

$u = \tan(\theta)$

$du = \sec^2(\theta)\, d\theta$

$u = 1, \quad \theta = \pi/4$

$u = \sqrt{3}, \quad \theta = \pi/3$

$$\int_{\pi/4}^{\pi/3} \frac{\sec^2(\theta)}{\sqrt{1+\tan^2\theta}}\, d\theta$$

$$= \int_{\pi/4}^{\pi/3} \frac{\sec^2(\theta)}{\sqrt{\sec^2\theta}}\, d\theta$$

$$= \int_{\pi/4}^{\pi/3} \sec(\theta)\, d\theta$$

$$= \ln|\sec(\theta)+\tan(\theta)| \Big|_{\pi/4}^{\pi/3}$$

$$= \ln(2+\sqrt{3})$$

$$- \ln(\sqrt{2}+1)$$

$$\int \sec(\theta) = \int \frac{1}{\cos(\theta)} \, d\theta$$

$$= \int \frac{\cos(\theta)}{\cos^2(\theta)} \, d\theta$$

$$= \int \frac{\cos(\theta)}{1-\sin^2(\theta)} \, d\theta$$

$u = \sin(\theta)$, $du = \cos(\theta)$

$$\int \frac{du}{1-u^2}$$

$$= \int \frac{1}{(1+u)(1-u)} \, du$$

$$\frac{1}{(1+u)(1-u)} = \frac{A}{1+u} + \frac{B}{1-u}$$

$$1 = A(1-u) + B(1+u)$$

$u = 1,\quad B = 1/2$

$u = -1,\quad A = 1/2$

$$1/2 \int \frac{1}{1+u} + \frac{1}{1-u}$$

$$= 1/2 \left(\ln|1+u| + \ln|1-u| \right)$$

$$= 1/2 \left(\ln|1+\sin(\theta)| + \ln|1-\sin(\theta)| \right)$$

$$\int_0^\infty e^{-\omega t}\,dt$$

$$= \lim_{x\to\infty} \int_0^x e^{-\omega t}\,dt \qquad \omega > 0$$

$$= \lim_{x\to\infty} \left.\frac{e^{-\omega t}}{-\omega}\right|_0^x$$

$$= \lim_{x\to\infty} \left(\frac{1}{\omega} - \frac{e^{-\omega x}}{\omega}\right)$$

$$= \frac{1}{\omega}$$