Name:

Math 116 Exam 3

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations up to four decimal points are acceptable unless otherwise indicated.

1) (10 points, 2 points each) True/False. If the sentence is false, correct the error. No justification is necessary.

a) If (r, θ) is in polar coordinates, then in rectangular coordinates, the y-coordinate of (r, θ) is $y = r\cos(\theta)$.

b) If $x'(t), y'(t)$ are defined and continuous on $[a, b]$, then the surface area obtained by revolving the parametric curve $\langle x(t), y(t)\rangle$ about the y-axis from $t = a$ to $t = b$ is defined as $\displaystyle\int_a^b 2\pi y(t)\sqrt{(y'(t))^2 + (x'(t))^2}\ dt$.

c) Any function $y = f(x)$ may be represented as a parametric curve.

d) The area inside a polar curve $r = f(\theta)$ from $\theta = \theta_0$ to $\theta = \theta_1$ is given by $\displaystyle\int_{\theta_0}^{\theta_1} [f(\theta)]^2\ d\theta$.

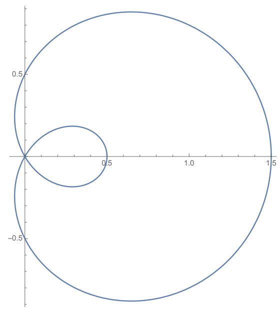
e) The curves $f(t) = \langle \sin(t), \cos(t)\rangle$ and $g(t) = \langle -\cos(6t), \sin(6t)\rangle$ have the same graph.

2) a) (6 points) What are the rectangular (Cartesian) coordinates of the polar point $\left(21, -\frac{4\pi}{3}\right)$?

b) (6 points) What is a representation in polar coordinates of the rectangular point $(0, -21)$?

3) (15 points) Determine the equation of the tangent line to the parametric curve given by $f(t) = \langle \arctan(7t), 8^{14t} \rangle$ at the point $t = 1/7$.

4) Consider the region in the 1st quadrant inside the inner loop of the curve $r = 1/2 + \cos(\theta)$. See the graph below.



a) (3 points) Shade the indicated region.

b) (6 points) Find the θ values determining this region.

c) (13 points) Find the area of the region.

5) Let $f(t) = \langle \sin(\pi \sin(t)), \cos(\pi \sin(t)) \rangle$.

a) (8 points) Set up an equation for the arclength of the graph of f from $t = 0$ to $t = \pi/6$

b) (13 points) Find the arclength of the portion of the curve described in part a).

6) Let $f(x) = x^2 + 12$.

a) (10 points) Set up an integral that represents the surface area obtained by revolving the graph of f from $x = 1$ to $x = 2$ about the y-axis.

b) (10 points) Determine the surface area obtained by revolving the graph of f from $x = 1$ to $x = 2$ about the y-axis.

BONUS: (10 points) Suppose f is a real-valued function and f' is continuous on $[a, \infty)$. Show that for any such f, if we define $g(s)$ to be the arclength of the graph of f from $x = a$ to $x = s$ $(a \leq s)$, then g is invertible.